Product Failure Recognition Via Comparison Of Sequential and Quickest Detection Algorithms

by

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A thesis submitted to the School of Engineering in partial fulfillment of the requirements for the degree of
Master of Science in Electrical Engineering

University of North Florida
College of Computing Engineering and Construction

March 26, 2021

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ACKNOWLEDGEMENTS

The fact that I am afforded the opportunity to the work presented in this thesis and thank those that helped me get where I am today is a privilege. I would not have made these successes without the unending support I have received from my family, friends, co-workers, and scholarly advisors. I would like to thank each and every one of you, but am unable to express my thanks in few enough words to fit within the confines allotted. So, I will simply say thank you for supporting me through this journey. Thank you for picking me up, and dusting me off when the research aspects became too great and I feared the idea of continuing. Thank you to all who listened to my numerous one-sided discussions which allowed me to verbalize and process aspects of my work. I would not be sharing this accomplishment without the community of amazing people who surround me everyday continuing to support me through each and every endeavor.

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ABSTRACT

Under similar conditions, products that are designed and used for similar tasks fail similarly. Developers may become aware of various product failure modes during the initial stages of new product generation, where redesign and failure mitigation processes can occur with minimal detriment to consumer safety. Developers strive to mitigate the potential for catastrophic failures. This thesis concentrates on when these failures occur outside of controlled conditions, specifically where the development of processes feature low accuracy sensing techniques that impact the safety and operation of the end user.

This thesis develops a set of statistical analysis simulation techniques using two existing methods: Sequential Analysis and Quickest Detection. Through the comparison of method-specific features, this thesis aims to assist future researchers unfamiliar with these methods to understand the individual characteristics of each as they pertain to failure mitigation. Each detection method is subjected to investigation via a pair of sensor models, a *strong sensor* and a *weak sensor*. Variable detection settings are used to quantify the operational characteristics of these sensors and their individual means of analysis. This thesis then compares both statistical techniques to recognize their overall usefulness to the topic of product failure analysis and mitigation pertaining to lower accuracy sensing processes that require longer sampling periods for better informed decisions. It is ascertained that the Sequential Analysis technique is best used when the initial system state is not yet known to the observer. The Quickest Detection method should be utilized when the initial state of a system is known and it is imperative to detect, with minimal delay, the occurrence of a random change-point in the operational status of the system.

Chapter 1

Introduction

Research shows that products designed for similar tasks, operating under similar conditions, fail similarly. Failure analysis is a core component of the product design process [Robert B. Stone and Wie, 2003]. Given this information, designers can incorporate efforts to minimize the potential of catastrophic operational failure once a product leaves their control [Robert B. Stone and Stock, 2005]. Faults can become known during initial testing schemes where the eventual breakdown is observed and controlled. It is when failure occurs in the operational environment, under little to no observation, that injury and loss of property creates more serious problems. Inevitably, products will fail. The aim of this research is not to prevent product failure, but to assist in the mitigation of operational failures during the early stages of development. These steps toward mitigation can be defined as improvements in the safety of product utilization as further implementation of failure detection techniques are applied during the development process.

This thesis compares a set of known statistical methods, Sequential Analysis [Wald, 2004] and Quickest Detection [Poor and Hadjiliadis, 2009], that are currently being used by researchers in the development of product failure mitigation techniques. It will investigate each method separately to start, followed by a comparative analysis of each to examine the differences in their setup and operating processes. Through the use of a literature review, comprised primarily of recently conducted work for each method and the theories they are based on, the existence of such a study is not readily available. It is believed that such a study will aid future research efforts in the development of new mitigation techniques using these statistical methods.

1.1 Product Failure

This research stems from an interest in the operation, eventual degradation, and failure of everyday products. This idea embodies the human drive to uncover existing problems in the world and discover solutions for them. It is this same drive that motivates much of the modern-day research to develop increasingly more accurate methods to mitigate the catastrophic failure of devices and systems in the everyday environment. This thesis specifically seeks to investigate and utilize the theories of Hypothesis Testing [Ronald E. Walpole and Ye, 2017] and Change-Point Detection [Aminikhanghahi and Cook, 2017], similar but different statistical methods by which algorithms for detecting system state changes (product failures) from repeated measurements can be cultivated.

Product/process failure analysis allows developers to analyze issues with a device prior to and during deployment into its intended environment. Process analysis has taken many forms in recent decades. Some of the most notable emerged during World War II, including Abraham Wald's development of Sequential Analysis [Gale, 2008], Alan Turing's Banburismus technique [Gladwin, 1997], and George Barnard's Optimal Stopping process. Each of these methods has been correlated to the production and movement of products and services into the modern era. It is important to realize that a product must go through a development process before being implemented into its final operational environment [Robert B. Stone and Stock, 2005].

In the fields of engineering and the sciences, the analysis of an existing problem and the development of a new product often can be accomplished in a similar manner. Both must be examined to determine key elements which must be addressed in future designs. A typical product development style is remarkably similar to the scientific method. It includes a stage for determining the problem, researching previous works, the development of potential solutions, and the production of trial solutions.

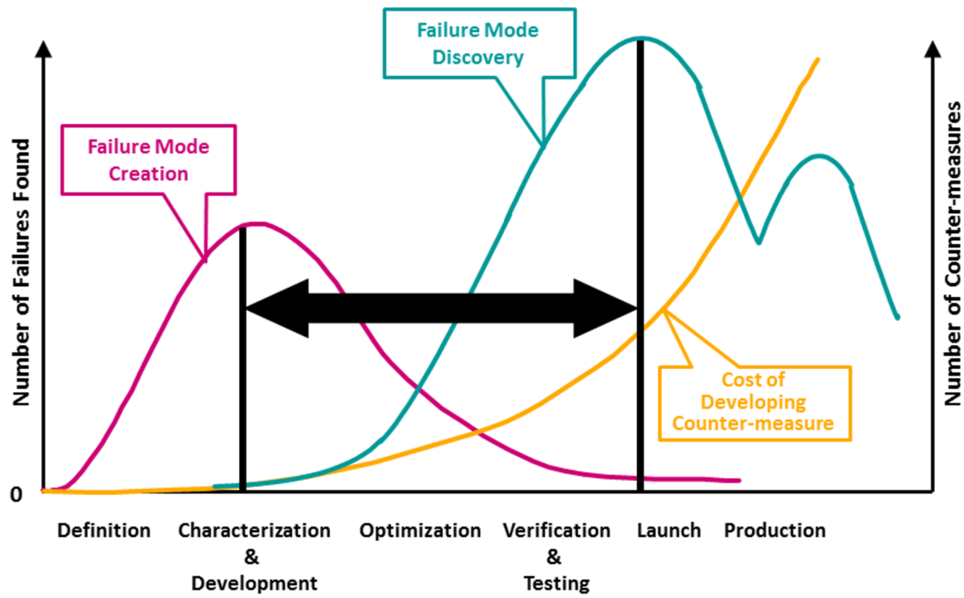

Figure 1.1: Late detection of product failure may lead to unsafe products being launched and expensive product redesign measures [Quality-One, 2020].

As Figure 1.1 illustrates, unknown failure modes may occur in early stages of development and go completely unrecognized until late into the production process, which may lead to expensive product redesigns and potential consumer safety issues. A failure mode is a mechanism by which a product ultimately breaks down into a state where it is rendered inoperable by normal standards. When a product under development is ready for initial trials in the marketplace, that product has already undergone a rigorous analysis process that typically unearths the most prevalent failure modes [Williams, 2015]. However, further testing may be required to evaluate, assess, and implement solutions into final design. Once the trial results are determined to satisfactorily solve the initial problem statement, the product can enter final production. The development of techniques to detect product failures early, minimize the number of redesigns, and mitigate potentially unsafe products releasing to the end user is the primary focus of many researchers [Robert B. Stone and Stock, 2005], [Robert B. Stone and Wie, 2003], [Braglia, 2000]. Additional analysis after a product is released into the market would allow developers to continue to gather data on additional failure modes that may have originally been unknown [Williams, 2015].

A product with sufficient capability of running automated diagnostics can determine the fidelity of its own operational state at any given moment [Pan and Zheng, 2020]. For example, if a product is originally designed with an optical sensor that, when activated, triggers a response which continues until intended deactivation through its own diagnostic processes, the product could determine if said sensor is functioning properly and notify the user otherwise. This integration of an automated monitoring system would support early detection of issues that are not noticeable to the user until the product fails in a more permanent manner.

Development of an automated process to continuously gather and analyze data on an implemented product was the overarching inspiration of this research study. Evaluation processes that may be implemented into future product designs would assist in the mitigation of the loss of property and lessen the possible impact to quality of life [Pan and Zheng, 2020]. As such, the originally proposed research which was to include the development of a series of algorithms that would utilize original adaptations of the aforementioned statistical analysis techniques. Through the evolution of this thesis study, it was determined to be more pertinent to analyze and demonstrate similarities and differences between these statistical methods, rather than evaluating each independently.

1.2 Thesis Contributions

In the practice of product design, the current standpoint to mitigate product failures is through the implementation of high-accuracy sensors and components that have high operational success characteristics [Leon, 2011]. High accuracy sensors possess high signal-to-noise ratios (SNR), which can be leveraged to make detection decisions based on a minimal number of samples that need to be collected prior to such a determination on the system state. Determinations of system state through the use of high SNR sensors is relatively straightforward, as only a handful of observations are near-conclusive. It is when a developer does not have access to high accuracy sensors that the determination process will require a larger number of samples to arrive at conclusive decisions; this thesis study concentrates on such low-SNR circumstances.

An existing product that exemplifies the roles of sensor accuracy and operational failure is the automated water faucet. This product is designed to detect a “customers” hands beneath a sensor that runs water when said hands are detected, then ceasing to run water when the hands are removed. There exists three instances of operation that automated water faucets may exhibit when a “customer” places their hands beneath the sensor: (i) normal operation, (ii) no operation, and (iii) operation which continues after the hands leave the sensor region. It is understood that the failure of these types of products is considered low liability, where the worst instance that occurs is a waste of water or customer frustration. This thesis demonstrates a series of statistical analysis processes, Sequential Analysis and Quickest Detection, which can be used to assist in recognizing the failure of products similar to this water faucet problem.

The analysis techniques of this thesis permit researchers with minimal exposure to the use and operation of the aforementioned statistical methods to implement these techniques sooner into their own research work. Furthermore, the outlined research provides future studies an analytical comparison of these two statistical methods from a common viewpoint.

1.3 Thesis Organization

The remainder of this thesis presents the investigation of the aforementioned statistical techniques, Sequential Analysis and Quickest Detection. The similarities and differences that exist between the two statistical techniques will be discussed and explored. This thesis is organized in the following manner: the **Background** section will illustrate the key aspects that make up each statistical method including how they are intended to function. The **Statistical Methods Development and Comparative Analysis** section will explore the developmental work of the two original algorithms for both techniques that were analyzed, and investigate the ultimate scheme that makes both methods unique. Finally, the **Recommendations and Conclusions** section will investigate the suggested future work with these statistical analysis methods, leveraging the discovered data from this thesis

Chapter 2

Background

The Sequential Analysis and Quickest Detection methods belong to the testing process theories of Hypothesis Testing and Change-Point Detection, respectively. To start, the importance of these two statistical theories will be illustrated, those being Hypothesis Testing and Change-Point Detection, and how they relate to the statistical methods this study plans to explore. Hypothesis Testing is the primary explanation method for the Sequential Analysis technique, while Change-Point Detection governs how Quickest Detection operates. These concepts will be expanded further upon throughout this chapter. The following sections will illustrate the predominant theories that drive how each of these statistical methods operate. Additionally, exploration of the initial development of select recent research works regarding utilization of these methodologies will be conducted.

2.1 Theory of Hypothesis Testing

The primary method that this work stems from is a sub-theory of the broader field of multiple/simultaneous inference, that being the theory of Hypothesis Testing [Shaffer, 1996]. Hypothesis Testing is used to determine how a researcher working with given data-sets can interpret their test results [Ronald E. Walpole and Ye, 2017]. As [Ronald E. Walpole and Ye, 2017] suggests, the main difficulty researchers must overcome with their data is not the data itself, but rather the formation of conclusions through the analysis of said data.

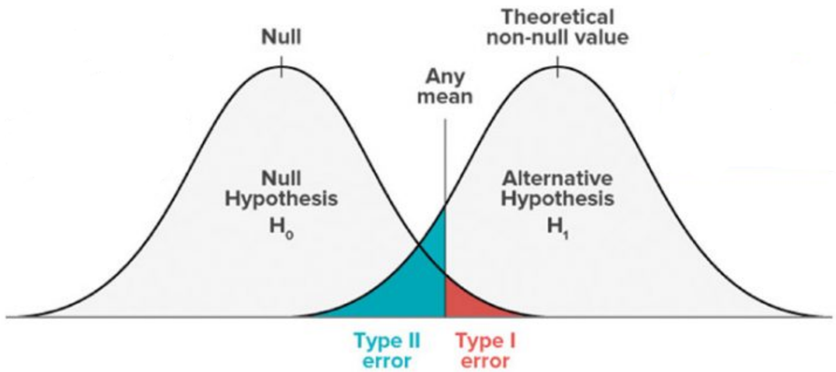

Figure 2.1: Error can occur when performing Hypothesis Testing [Data-Science, 2020].

As Figure 2.1 illustrates, when performing a hypothesis test on a given data-set, researchers postulate a statistical hypothesis (null hypothesis) on an aspect of the data-set that informs on the overall experimental outcome from the initial data collection process [Ronald E. Walpole and Ye, 2017]. At the conclusion of the statistical hypothesis test, determining how true or false the results are can be difficult due to the fact that only a segment of the population is typically analyzed [Ronald E. Walpole and Ye, 2017]. Since absolute certainty is not guaranteed, resulting opinions are formed based on evidence supported by the data analyzed. A potential consequence of this limitation is an incorrect conclusion regarding the data. The probability of an incorrect conclusion can be categorized as a type I or type II error [Ronald E. Walpole and Ye, 2017] [Shaffer, 1996] [Wald, 2004].

Figure 2.1 shows the means by which type I and type II errors occur. Type I errors refer to a rejection of the null hypothesis when it is in-fact correct, also known as a “false positive” conclusion [Ronald E. Walpole and Ye, 2017]. In regards to the overarching scope of product failure analysis, a type I error occurs when a progressively failing/already failed product is determined to still be functioning properly. An example of a type I error can be illustrated by the automated water faucet problem where the sensor detects the presence of a “customers” hands and begins running water. When the hands are removed the sensing unit continues to determine the need for water exists which leads to the faucet continuing to run. In this instance the sensing unit falsely determines the "customers" hands were still present and therefore continued supplying water. Conversely, type II errors refer to the occurrence of a "false negative" conclusion [Ronald E. Walpole and Ye, 2017]. Such errors occur in products which are currently functioning within normal parameters, but for some reason are deemed to be failing/already failed. A type II error occurrence can be demonstrated by the automated water faucet problem again. Specifically, when a "customer" places their hands below the sensing unit and no water begins to run. The sensing unit incorrectly determined there to be no hands present.

The analysis and formation of conclusions regarding an unknown distribution data-set can be realized through the use of Hypothesis Testing [Wald, 2004]. It is important to note that typically unknown distributions are not entirely unknown, i.e. some knowledge about the distribution can be understood through the use of the *a priori* information gathered prior to the initiation of the testing process [Wald, 2004]. Additionally, as [Wald, 2004] states, "*The functional form of the distribution function is known and merely the values of a finite number of parameters involved in the distribution function are unknown*". With this understanding, development for the general approach of Hypothesis Testing can be begin. As [Wald, 2004] suggests, one may view their test procedures as a division of the samples collected using an online process, respectively. An online process analyzes collected samples in close to real-time and produces results as quickly as possible to allow for sampling updates and system changes to be enacted. Conversely, offline processes analyze complete data-sets that were collected at a date and time prior and develop results that may be used in a later occurring, similar instance. One can use observations of the sample divisions to create concluding decision rules for their test procedure. Throughout the creation of an experimental procedure, it is highly important to set decision rules with the end goal of minimizing type I and type II errors [Wald, 2004].

2.2 Theory of Change-Point Detection

A modified approach of the theory of Hypothesis Testing, the Change-Point Detection method attempts to detect a change in a system model with fixed or minimal delay as well as minimize the frequency of false alarms [Rudolf B. Blazek and Tartakovsky, 2001] [Basseville, 1988]. False alarms from detection processes can refer to the occurrence of type I (false positives) and type II (false negative) errors. Additionally, Change-Point Detection is related to a technique known as Change-Point Estimation/Change Point Mining [Aminikhanghahi and Cook, 2017]. The main difference between Change-Point Detection and Change-Point Estimation is that Change-Point Estimation works to determine known change-points in time series data, while Change-Point Detection measures when actual change-points occur [Aminikhanghahi and Cook, 2017]. A change-point is the time instance that the state of the system being observed changes from one data distribution to another [Alexander G. Tartakovsky and Sokolov, 2013]. An example of a change-point occurrence can be described by the automated water faucet problem. The sensing unit for this device is continually sampling the environment located below it for the presence of hands. When the unit detects the presence of hands, it begins to run water. Change-Point Detection is tasked to identify abrupt/unexpected changes in time series data from a sampled norm [Geng and Lai, 2013], then determine if this variation is due to the occurrence of a change-point in the data stream or if it is noise from the testing environment [Shilpy Sharma and Obimbo, 2016]. Furthermore, change-point algorithms can be developed to detect single or multiple changes in a system or data-set [Shilpy Sharma and Obimbo, 2016]. Figure 2.2 shows an example of Change-Point Detection. The occurrence of a change-point can be observed occurring at time t = 200 and at time t = 400 the detection process discovers the change. The detection technique would be tasked with determining the point in time when the change occurred with minimal delay.

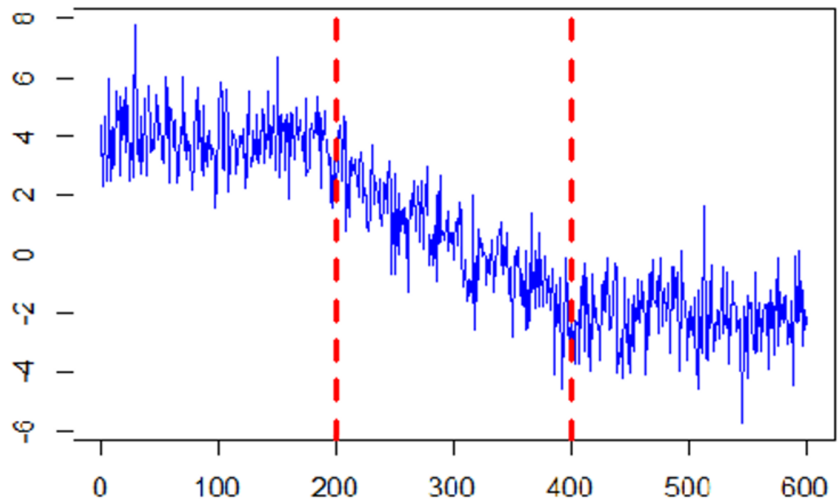

Figure 2.2: Change-Point Detection operation and results [Killick, 2014].

The information pertaining to this study will primarily concern the categorical Quickest Detection algorithm type to be developed and utilized for this thesis study. Throughout this section, the use of existing survey references to the topic will be used to develop the relevant techniques generated during the creation of this thesis. These surveys will be leveraged to support out-of-scope methods for the remaining breadth of the topic of Change-Point Detection on their own. A brief summary of the operational use and existing techniques for Change-Point Detection will be illustrated here.

When a change-point is detected in a system the test sequence will make a decision that stops and classifies the test [Poor and Hadjiliadis, 2009]. This method of decision making and classification is the technique utilized to develop Quickest Detection. Quickest Detection methods continually sample, analyze, and make decision(s) based on the previous and current collected data from the sensor model [Poor and Hadjiliadis, 2009]. Additionally, the Quickest Detection process to be utilized belongs to the class of unsupervised observation techniques. This is due to its use of the likelihood ratio analysis, where the probability density of two consecutive time intervals are the same if they belong to the same state [Aminikhanghahi and Cook, 2017]. This algorithm can be described as a single change-point, online, non-parametric, unsupervised, filtering process.

Figure 2.3 illustrates the characteristics of Change-Point Detection that the Quickest Detection formulation represents.

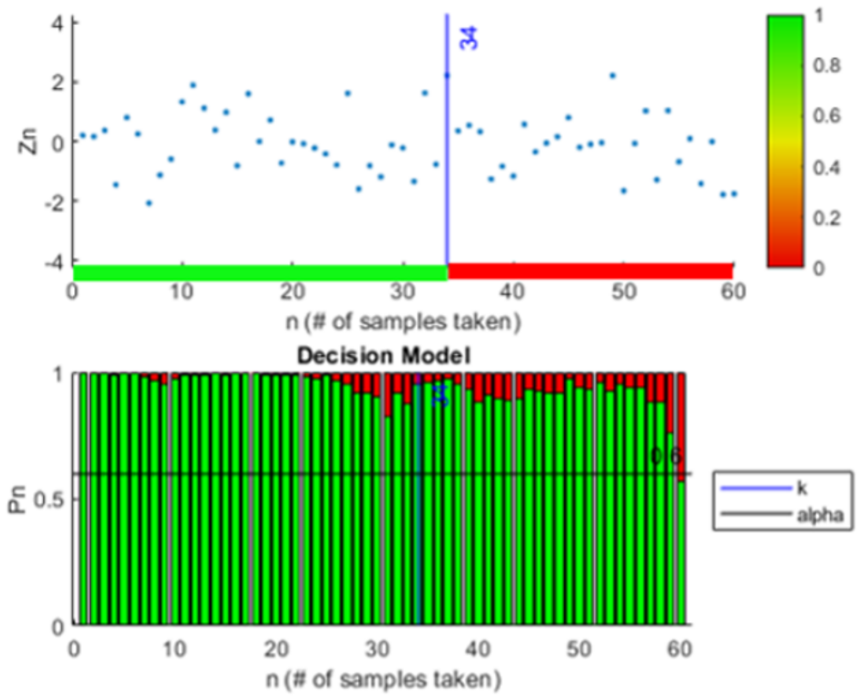

Figure 2.3: Example of Change-Point Detection through the use of the Quickest Detection formulation.

As [Basseville, 1988] shows, this technique can be observed through its use in the following example industries: speech recognition, geophysical and seismic sciences, biomedical signals processing, aeronautics, vibration monitoring, and many more. [Richard J. Radke and Roysam, 2005], [Basseville, 1988], and [Aminikhanghahi and Cook, 2017] describe Change-Point Detection in a much deeper level than this research document requires due to the usefulness such a technique affords to many fields.

Change-Point Detection algorithms can be further divided into two classes: smoothing and filtering [Sile Hu and Chen, 2018] [Chopin, 2007]. Smoothing techniques are typically utilized when the data-set is analyzed via offline methods, i.e. the collection process is finished and decisions are made at a later time [Chopin, 2007]. Furthermore, the complete data-set is used for estimating the occurrence and location of all change points within [Chopin, 2007]. Filtering processes are those which occur as data is being collected and stored, known as an online analysis process [Sile Hu and Chen, 2018]. Filtering is classified as a Monte Carlo method since it relies on continued sampling of the data-set to obtain its numerical results [Sile Hu and Chen, 2018]. Figure 2.3 illustrates the filtering process as the blue sample collection values shown in the Z_n graph.

As [Aminikhanghahi and Cook, 2017] demonstrates, detection techniques can be supervised or unsupervised in the nature of their use. There are a series of methods and developed techniques that may be used when performing Change-Point Detection as [Shilpy Sharma and Obimbo, 2016], [Richard J. Radke and Roysam, 2005], and [Aminikhanghahi and Cook, 2017] illustrate. The most common techniques for supervised machine learning algorithms include: decision tree, naive Bayes, Bayesian net, support vector machine, nearest neighbor, hidden Markov model, conditional random field, and Gaussian mixture modeling [Aminikhanghahi and Cook, 2017]. These methods utilize the implementation of a machine learning process. This research study does not devise the use of a machine learning process to this point. Thus, it was necessary to instead turn to unsupervised data collection. Methods of unsupervised data collection include: likelihood ratios, subspace modelling, probabilistic methods, kernel-based observations, graph based techniques, and clustering methods [Aminikhanghahi and Cook, 2017]. The likelihood ratio approach is the most widely used statistical method when performing unsupervised Change-Point Detection analysis [Basseville, 1988]. Furthermore, the likelihood ratio method analyzes probability distribution data collected before and after potential change points and identifies a change point, if the two time sequences significantly differ in distribution categorization [Aminikhanghahi and Cook, 2017]. Additionally, Figure 2.3 illustrates the use of the probability likelihood ratio by associating each of the collected samples (shown as blue data points on the Z_n graph) to the colormap shown to the right of the Z_n graph. This association is then used to update the probability likelihood ratio, P_n graph, until the stopping decision is satisfied/crossed (the horizontal black line on the P_n graph).

The use of Trend Analysis can be incorporated into a Change-Point Detection process [Shilpy Sharma and Obimbo, 2016]. Trends in gathered data can be analyzed for determining gradual changes in the future from previously collected data, and are estimated through the use of parametric and non-parametric techniques [Shilpy Sharma and Obimbo, 2016]. A parametric approach estimates any unknown data parameters based on the training data after instructed assumptions on the form of the function are determined. Conversely, non-parametric techniques make no such assumptions on the form of the function being analyzed [Aminikhanghahi and Cook, 2017].

There exist a multitude of performance metrics that a Change-Point Detection technique can be measured against. These can range from accuracy and precision measures, sensitivity, receiver operating characteristics curves, and precision-recall curves [Aminikhanghahi and Cook, 2017]. For the purposes of the work developed within this thesis, a difference in time between the actual change-point and its detection can operate as a sufficient performance metric when the previously mentioned metrics are not appropriate [Aminikhanghahi and Cook, 2017].

2.3 Literature Review: Sequential Analysis

The first idea for a sequential test goes back to H.F. Dodge and H.G. Romig who proposed a double sampling inspection procedure [Dodge and Romig, 1929] [Wald, 1945]. A double sampling scheme decides if another sample should be taken, and is dependent on the result from the previous sample [Gale, 2008]. Another researcher who continued developing quality control methods was Walter A. Shewhart. Shewhart worked with problems of quality control which led to the creation of modern Sequential Analysis, developed by Wald [Shewhart, 2017] [Siegmund, 2003] [Lai, 2001]. In 1943, Walter Bartky produced a generalized idea based on the work previously conducted by Dodge and Romig, called Multiple Sampling [Gale, 2008]. Multiple Sampling was related to the work for the United States government conducted by Abraham Wald during World War II [Gale, 2008]. Sequential Analysis was further developed and later published in 1943 by Abraham Wald, Jacob Wolfowitz, W. Allen Wallis, and Milton Friedman as the Sequential Probability Ratio Test (SPRT), while at Columbia University's Statistical Research Group. Due to its usefulness in the area of Hypothesis Testing, Sequential Analysis was used as a tool for more efficient industrial quality control during the Second World War [Gale, 2008] [Lai, 2001]. It was designated with a restricted classification by the United States government due to its immense value to the ongoing war effort. The cumulative research from Abraham Wald, Alan Turing, George Barnard, and many other groups led to a series of opportunities after the Second World War that furthered the topic of Sequential Analysis and other similar statistical methods [Siegmund, 2003]. As [Qiyue Zou and Sayed, 2010] suggests, [Poor and Hadjiliadis, 2009] is a wealth of information on the progression of research conducted in the topic of Sequential Analysis post-World War II up to the near modern day. Furthermore, [Lai, 2001] and additional surveys can be utilized to continue to develop understanding of the breadth of conducted research into the modern day topic of Sequential Analysis.

Sequential Analysis is a statistical analysis method that leverages the use of *a priori* distribution information and then continues to operate by the *a posteriori* technique [Sochman and Matas, 2005] [Rudolf B. Blazek and Tartakovsky, 2001]. The information gained *a posteriori* allows it to determine whether or not to stop a testing sequence based on previous and current observation data. [Rudolf B. Blazek and Tartakovsky, 2001]. Furthermore, Sequential Analysis allows for variable observation sizes per test until a stopping rule is satisfied [Wald, 2004].

Figure 2.4 visually demonstrates how the Sequential Analysis statistical method is a Hypothesis Testing technique where the initial state is unknown. Only after incoming samples are analyzed and one of the stopping thresholds is satisfied will the process continue [Wald, 1945]. The most effective use of this testing method occurs when there are sequential observations that become available for analysis with minimal delay periods in between [Gale, 2008]. The stopping rules are typically determined prior to the initiation of the test, and the test concludes only after one of these stopping conditions are achieved [Wald, 1945]. During the operation of a Sequential Analysis test, one of three basic statistical test decisions is made: (i) to accept the hypothesis being tested (known as the null hypothesis), (ii) to reject the null hypothesis or, (iii) to continue the experiment by making an additional observation [Wald, 1945].

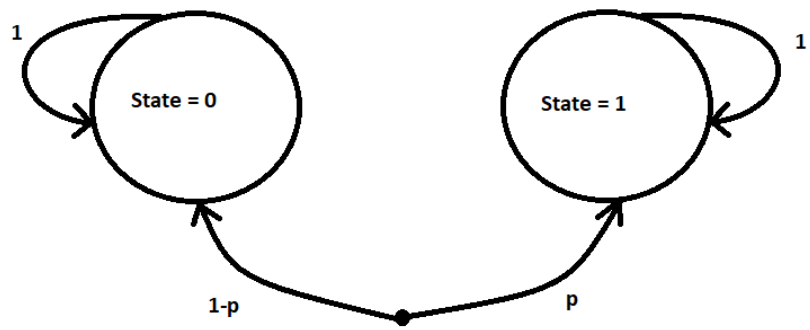

Figure 2.4: Markov chain for Sequential Analysis, initial state of system is unknown until sample collection yields significant likelihood understanding.

Figure 2.5 illustrates a test process utilizing the Sequential Analysis detection method.

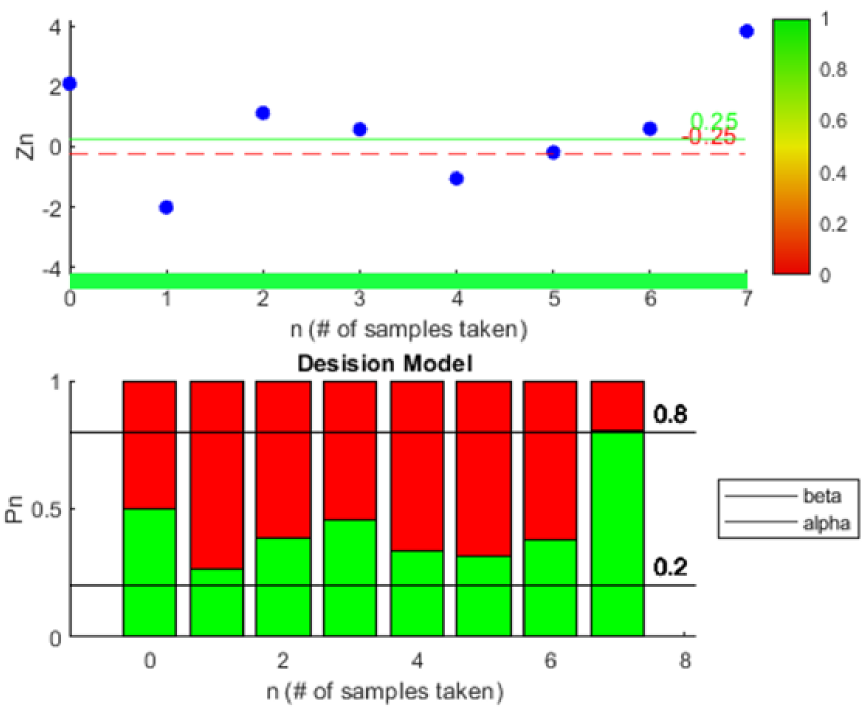

Figure 2.5: Example of Sequential Analysis method, initial state is unknown until significant amount of sample collection and analysis occurs.

As Figure 2.5 shows, the test sequence begins to collect data samples directly from the sensor model (shown as the blue data points on the Z_n graph). These data values are then associated to the colormap to the right of the Z_n graph which informs the decision model on how to update the probability likelihood ratio shown in the P_n graph. Furthermore, the P_n graph begins the testing sequence with a likelihood ratio of 0.5 due to the nature of the Sequential Analysis technique not possessing information on the exact state of the system being observed at the tests initiation. It is only after sufficient sample collection has occurred that the detection process can make a decision on the system state which then ends the test sequence.

As seen when using this method, there are two stopping thresholds: α and β. For the formulation of this technique presented in this thesis, α is defined prior to initiation of testing, and β is determined by Equation 2.1:

$$\beta = 1 - \alpha \tag{2.1}$$

The alpha stopping threshold will symbolize the null hypothesis distribution (the product is failing or has failed) and the beta threshold will be for determining the alternative hypothesis (the product is currently functional). Further expansion on this test setup and the results will be illustrated in Chapter 3 of this thesis.

The use of Sequential Analysis in research based applications has yielded a number of useful results in the fields of engineering, education, mathematics/statistics. During test sequences of the SPRT style algorithm, a pattern can be observed relating to the value of the probability density function being calculated at each time interval [Jeong, 2003]. This pattern may be used to determine a faster solution to the test sequence through a process that calculates the individual stopping threshold for each test sequence without the need for user defined thresholds. A similar technique was used to develop a Discussion Analysis Tool (DAT) in 2003 [Jeong, 2003]. The DAT uses Sequential Analysis to identify patterns in interactions and determine which interactions promoted critical thinking. This was done by computing probabilities and converting them into normalized significance scores that showed the progress of critical thinking in student interactions [Jeong, 2003]. When the algorithm recognized a pattern of the overarching distribution of the test sequence, it calculated the solution to the test in a reduced delay time [Qiyue Zou and Sayed, 2010].

A similar method of classifying the distribution of tests, known as WaldBoost, was developed by using a trained strong classifier H_T with a set of known thresholds $\theta_A^{(t)}$ and $\theta_B^{(t)}$. In the event H_T exceeded a pre-set threshold then a decision based on a strong classifier would be made. If no threshold was met a weak classifier would be used [Sochman and Matas, 2005]. The use of a classifier similar to DAT would further decrease the amount of time delay during a testing sequence.

As [Bai and Gupta, 2017] suggests, the pre-determination of a test sequences cost function which is associated with various sensor outcomes should occur. As the cost function is adjusted for varying test sequences, more accurate solutions can be determined without exceeding defined cost values. In the event that a false negative occurs during the decision process of a test, the total cost would equal the time delay and the value from an error occurring. If a false negative were to occur, then the value defined for the cost of a false negative would be accrued in addition to the total delay cost from the testing sequence [Qiyue Zou and Sayed, 2010]. For example, if a test yields a delay of $n = 1150$ and a false negative occurs with a defined cost of 1000, then the total cost would be equal to $TotalCost = n + 1000 = 2150$. Therefore, this test would benefit from the use of an Optimal Stopping process, similar to DAT and WaldBoost, to minimize the cost of the test sequence.

Additional work being performed utilizing the SPRT method exists in the intrusion detection systems design. These systems can be classified as predominantly being either Signature Detection Systems or Anomaly Detection Systems [Rudolf B. Blazek and Tartakovsky, 2001]. The work presented in [Rudolf B. Blazek and Tartakovsky, 2001] is an example from the Anomaly Detection Systems class. These researchers sought to develop a method that yielded early detection of attacks from the class of “denial-of-service attacks” [Rudolf B. Blazek and Tartakovsky, 2001]. Their research stemmed from the Change-Point Detection problem, where it was tasked to detect a change in the test model, with fixed or minimal delays, and to control the occurrence of false detections (false alarms) [Rudolf B. Blazek and Tartakovsky, 2001]. Furthermore, [Rudolf B. Blazek and Tartakovsky, 2001] developed a multistage detection algorithm using the individual batch and sequential algorithms they were originally working with. These algorithms observed multiple layers of network protocol data and were tasked with detecting changes in the networks traffic that exhibited similar to known attack methods. Similar to Sequential Analysis and other Hypothesis Testing techniques, their algorithms used thresholds to trigger when the occurrence of an attack was observed and to minimize the rate of false alarms [Bai and Gupta, 2017]. Their multistage detection technique was found to be more robust and reliable at an acceptable increase of delay [Rudolf B. Blazek and Tartakovsky, 2001].

2.4 Literature Review: Quickest Detection

Quickest Detection is a statistical analysis method based on the theory of Change-Point Detection, also referred to as the Quickest Probability Ratio Test (QPRT) [Taragay Oskiper, 2005]. It is a generalization of Wald's Sequential Analysis technique, and therefore, is also categorized as a Hypothesis Testing method [Rudolf B. Blazek and Tartakovsky, 2001]. QPRT is used to detect changes that occur in a system at random time instances, and these changes are referred to as a shift from a "Normal" to an "Abnormal" operating state. [Alexander G. Tartakovsky and Sokolov, 2013] [Taposh Banerjee and Veeravalli, 2014]. Figure 2.6 illustrates the Markov chain process for a Quickest Detection technique where the initial state of the observable system is functional.

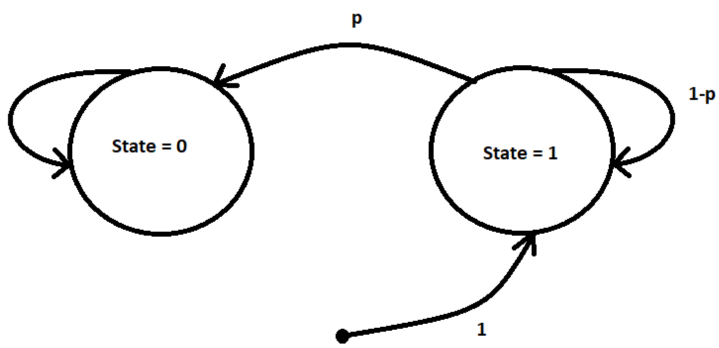

Figure 2.6: Markov chain representing Quickest Detection operation.

The general operation of the Quickest Detection method. The operating state of the system is known to be functioning when the testing process begins. It is only after an unknown amount of time that the state of the system changes from functioning to not functioning with probability p. Sampling continues until the detection method determines such a change has occurred and ceases the collection of samples. Figure 2.7 shows a test run from a Quickest Detection algorithm performing in this manner.

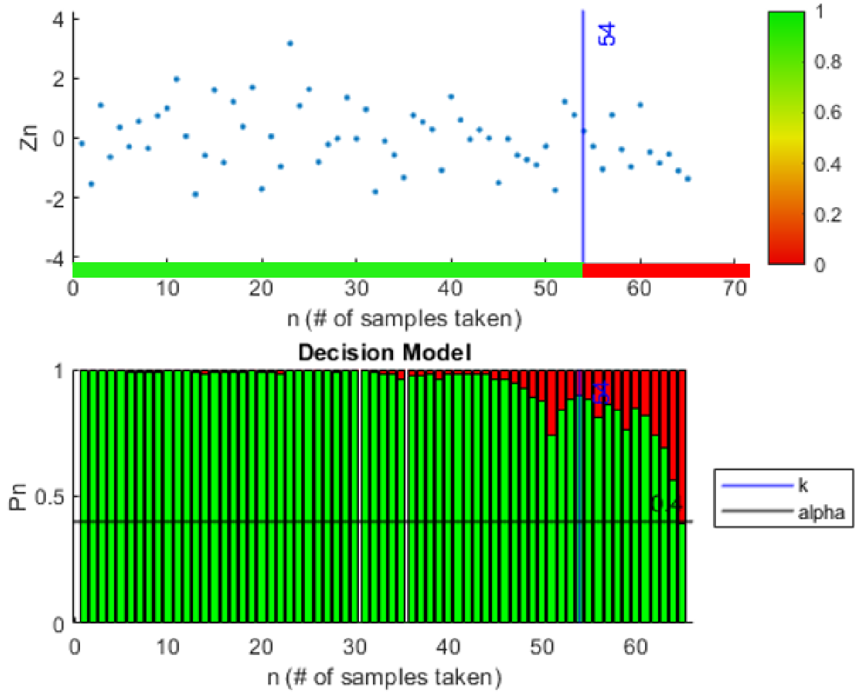

Figure 2.7: An example of the function of the Quickest Detection method.

The incoming data collection for an online Quickest Detection process (shown as the blue data points on the Z_n graph). These data points are then further analyzed and associated with the colormap to the right of the Z_n graph, similarly to the technique used by the Sequential Analysis method. The occurrence of a change-point is shown on both the Z_n and P_n graphs by the vertical blue line (shown at sample time instance n = 54). When observing the P_n graph (the probability likelihood ratio for the decision model), the incoming samples after the change-point occurrence significantly update the probability likelihood ratio until the stopping decision shown as the horizontal black line is satisfied/crossed. At which point the test sequence ends.

The Quickest Detection method is the optimization of two performance indices, the first being the delay between the time a change occurs and when it is detected, and the second being a measure of the frequency of false alarms [Poor and Hadjiliadis, 2009] [Rudolf B. Blazek and Tartakovsky, 2001]. As [Poor and Hadjiliadis, 2009] states, Quickest Detection is possible using both Bayesian and non-Bayesian approaches to the problem. Bayesian solutions include an unknown change-point that is assumed to be a random variable by a known prior distribution. In contrast, non-Bayesian solutions are traditionally formulated in the absence of the prior distribution being known, and thus involves averaging over the change-point distribution [Poor and Hadjiliadis, 2009].

Research into Quickest Detection methods surged during the Cold War era [Stephen R. Carpenter and Pace, 2014]. As nations saw a need to detect incoming threats, a method of detecting such threats early enough for an appropriate response could be mounted was needed. Because QPRT typically runs by developing a likelihood ratio of two models [Leian Chen and Wang, 2018], it could be adapted to meet these needs for detecting potential threats to national security [Stephen R. Carpenter and Pace, 2014]. The two model types that are utilized by the Quickest Detection method are the normal environment it was deployed into, and a randomly occurring change-point situation [Poor and Hadjiliadis, 2009] [Stephen R. Carpenter and Pace, 2014]. It is through the use of such a model-comparison scheme that QPRT methods minimize the time of detection of an occurring change-point and the rate of potential false alarms [Poor and Hadjiliadis, 2009].

The use of Quickest Detection algorithms in real-world development and simulated environments is much like that of Sequential Analysis, due to both being widely used in research and development applications [Lifeng Lai and Poor, 2008]. [Leian Chen and Wang, 2018] illustrates an example where the use of Quickest Detection, rather than monitoring via an experimental long-term model-based method, eliminates several elements of the testing scheme that would normally need to be accounted for. Other variations where Quickest Detection has been used recently to solve real-world issues include: automatic seizure detection [Sabato Santaniello and Sarma, 2011], detection in Smart Grid Networks [Yi Huang and Han, 2016], photovoltaic system faults [Leian Chen and Wang, 2018], changes to existing ecosystems [Stephen R. Carpenter and Pace, 2014], as well as military activities, industrial processes, and financial analysis [Stephen R. Carpenter and Pace, 2014]. For the purposes of this discussion, this thesis develops the tools required for use of the discrete-time Bayesian viewpoint, where an unknown change-point is treated as a random variable with a known prior distribution [Poor and Hadjiliadis, 2009]. Furthermore, a Quickest Detection algorithm differs from a Sequential Analysis algorithm in that it does not require prior knowledge of the change-point distribution or when it occurs in the time series of the testing sequence [Leian Chen and Wang, 2018] [Alexander G. Tartakovsky and Sokolov, 2013].

Without the development of additional Quickest Detection methods, changes in system data and ecosystems would only be analyzed a significant amount of time after a change has occurred [Stephen R. Carpenter and Pace, 2014]. The strongest benefit of the quickest-change method is its ability to immediately review data when it is sampled from the test environment [Stephen R. Carpenter and Pace, 2014]. This feature allows observers to detect approaching change-points before they occur [Taposh Banerjee and Veeravalli, 2014]. As impactful as this type of analysis can be, it is important to note that it is not always possible to detect change-points before they occur due to the sensitivity level of the test sequence stopping threshold [Stephen R. Carpenter and Pace, 2014]. In most instances, this should not disrupt effectiveness of the detection method. In a Quickest Detection algorithm, the change/fault that occurs persists until either the fault is addressed, the next change occurs, or the sample sequence is terminated [Taposh Banerjee and Veeravalli, 2014] [Taragay Oskiper, 2005] As discussed in section 2.3, Quickest Detection also utilizes the cutoff threshold system shown in Figure 2.7. This is due to Quickest Detection's origin as an adapted variation of Sequential Analysis [Poor and Hadjiliadis, 2009].

Examples of QPRT methods implemented in real-world based development include: [Sabato Santaniello and Sarma, 2011] developed a detection method for drug-resistant seizure state occurrence in patients and [Yi Huang and Han, 2016] proposed and designed a real-time detection algorithm that worked to quench the injection of false data into smart grid networks. The algorithm [Sabato Santaniello and Sarma, 2011] developed worked on three frameworks: construction of multi-channel intracranial EEG statistics to determine the state occurrence, a method to model the collected statistics, and the development of an optimal control-based Quickest Detection strategy. The researchers associated the term *dynamic detector* to their work, as it was able to evolve the classifier over time based on the totality of the collected data stream measurements [Sabato Santaniello and Sarma, 2011]. Furthermore, their algorithm worked on the basis of beginning the monitoring process and observing the patient data until a seizure had been detected, then terminating and restarting the method. The Quickest Detection method utilized in this study will be further developed in the next chapter and will operate similar to this form. After [Sabato Santaniello and Sarma, 2011] applied their framework to training data and four subject data-sets of subjects who exhibited drug-resistant seizures, they determined their method yielded 100% success rate on not only the training data but also the patient data. [Yi Huang and Han, 2016] sought to develop a detection method that observed and determined the interjection of false data into a smart grid network in real-time. Their method would compliment energy management systems in control centers by determining when malicious attacks were compromising the network [Yi Huang and Han, 2016].

[Poor and Hadjiliadis, 2009] instructed that the Quickest Detection method can be designed for Bayesian and non-Bayesian approaches, respectively. [Yi Huang and Han, 2016], utilizes a non-Bayesian format for their detection algorithm, with the understanding that threshold design introduces a trade-off between delay and detection error. Their formulation includes a technique known as *bad data detection* for smart grid applications, which allows for detection of adversary information quickly with a minimal decrease in accuracy [Yi Huang and Han, 2016]. Their proposed technique does not require the distribution attributed to attacks to be known prior to initiation [Yi Huang and Han, 2016]. Results collected by [Yi Huang and Han, 2016] illustrated that their detection algorithm was successful in detecting accurately and alarming with a minimal delay.

Chapter 3

Statistical Methods Development and Comparative Analysis

Proceeding from the information given in earlier sections, this thesis will further develop the original, independent statistical methodology; that being the Sequential Analysis and Quickest Detection algorithms used during this research. First, the sensor models that will be used during all testing events for both of the detection methods will be examined. There were two sensor models used, one *Strong sensor*, and one *Weak sensor*. Each statistical method will be evaluated individually, tasked with analyzing the two sensor models. Then a functional comparative analysis and examination of the results from each method will be systematically conducted. Furthermore, an investigation of the differences between each method's operational characteristics will occur.

3.1 Analysis Sensor Models

The two sensor models used in this thesis will represent a *strong sensor*, i.e. one with high precision, and a *weak sensor*, or one that has a low precision on measurements. The precision of each sensor model can be explained by the signal-to-noise ratio (SNR) each resembles. The *strong sensor* has a SNR of 1.5 and the *weak sensor* possesses a SNR of 0.5. Lower SNR values may be understood by static or noise on a signal, i.e. radio frequencies where the observer can hardly discern what is being transmitted due to high interference. The *strong* and *weak sensors* report their data via dual Gaussian Normal distribution models, where distribution mean values will be varied between each sensor. Furthermore, lower values of SNR are more difficult for the detection processes to differentiate between the two distributions that make up the sensor model set to be observed. The distributions that make up the sensor models will be centered about the zero value on the x-axis as Figures 3.1 and 3.2 illustrate.

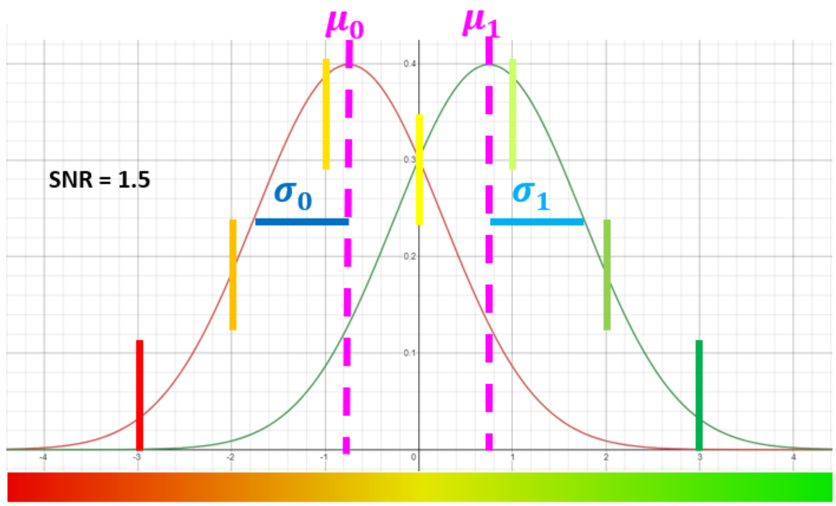

Figure 3.1: *Strong sensor* - Normal Gaussian distribution [Desmos, 2021].

As Figure 3.1 shows, the characteristics that represent the *strong sensor* distribution can be written as: the mean of the left (red) normal distribution (μ_0 = -0.75), the mean of the right (green) normal distribution (μ_1 = 0.75), and the standard deviation for both distributions ($\sigma_0 = \sigma_1$ = 1.0). With these distribution characteristics, the *strong sensor* allows for higher precision and a stronger ability for the detection processes to differentiate between similar measurements due to its higher SNR value of 1.5.

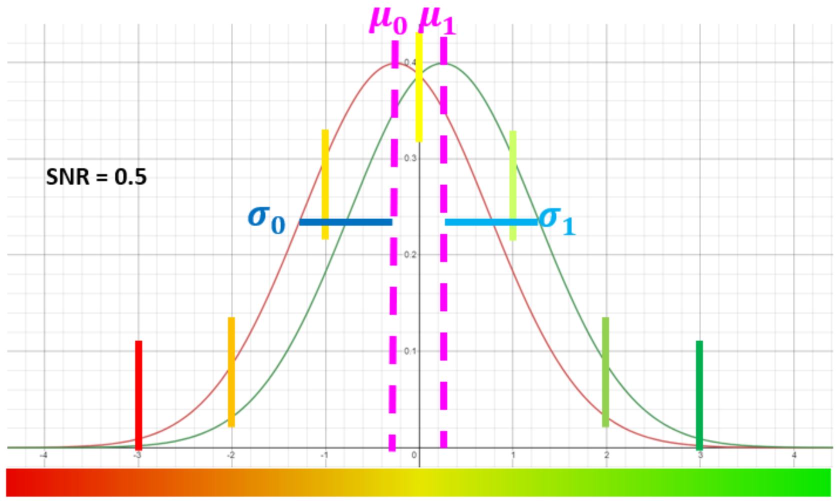

Figure 3.2: *Weak sensor* - Normal Gaussian distribution [Desmos, 2021].

Figure 3.2 shows similar graphical characteristics as Figure 3.1 with: μ_0 = -0.25, μ_1 = 0.25, σ_0 = σ_1 = 1.0. The weak sensor features distributions that are spaced much closer to each other when compared to the *strong sensor*. This spacing difference will test the precision of the detection methods and their ability to decide on the appropriate distribution when values are observed where the *weak sensor's* Normal distributions overlap heavily. Furthermore, the analysis processes will use various α-Knob values that dictate the decision region stopping thresholds, alpha and beta, during each test process. Specifically, the effect the variability of the α-Knob value has on the detection process by the way it changes the decision regions used to conclude each testing sequence. This will be discussed further later in this chapter when each of these analysis techniques are developed, as well as during the comparative analysis section.

3.2 Sequential Analysis Algorithms

From Section 2.3: the Sequential Analysis statistical method is a Hypothesis Testing technique that continues the testing process until one of the predetermined stopping thresholds are satisfied. Examination of the two algorithms developed during this thesis study in regard to the Sequential

Analysis method will take place. These include a single-shot algorithm that produces results for a pre-defined sensor model and a multiple-run varying α algorithm that will perform 10,000 test runs at a single value of α, before advancing to the next value of α and performing the same process until $\alpha \approx 1.0$.

3.2.1 Single-Shot Sequential Analysis

Sequential Analysis leverages a set of user specified initial conditions, predetermined by the test initiator, to acquire a prior stage measurement. After the prior stage measurement, the sensor model is known to the algorithm but not to the testing sequence. This information is stored for later use. A per-stage measurement is then taken and analyzed to determine if one of the stopping thresholds are met. If neither of the stopping thresholds are satisfied, another measurement is taken. Once a stopping threshold is satisfied, the process defines the distribution of the test run. After the distribution is defined, the total cost of the test is calculated via the type error which occurred during that test, as well as the cumulative per-stage cost. The type error is defined as no error, type I error (false positive), or type II error (false negative). Each of these three solutions yield a different impact over the total cost of the test as defined by the overall testing cost function set by the test initiator. The per stage cost can be calculated by the total number of times data was collected and multiplied by the cost attributed to continuing the testing sequence. The value associated with collecting another sample depends specifically on the overall detriment one may see to doing so. Both the per-stage cost and the type error encountered at the end of the test are then added to determine the total cost of the test. The use of individual test sequence cost values did not pertain to this thesis study when performing the sought after individual detection technique operational understanding and the functional comparison aspects.

In terms of the Sequential Analysis method (additionally referred to as the Sequential Probability Ratio Test (SPRT)), α is typically defined as the region of probability where a testing sequence continues to gather data. As the value of α approaches 0, the testing sequence will continue everywhere, and when α approaches 1, the sequence will stop everywhere. α can be further understood as the willingness the detection process has to collect another sample, when α approaches 0 the detection method is more willing to collect samples. When α approaches 1 the detection process is less willing to collect additional samples to leverage in the decision operation. Figure 3.3 illustrates this willingness concept.

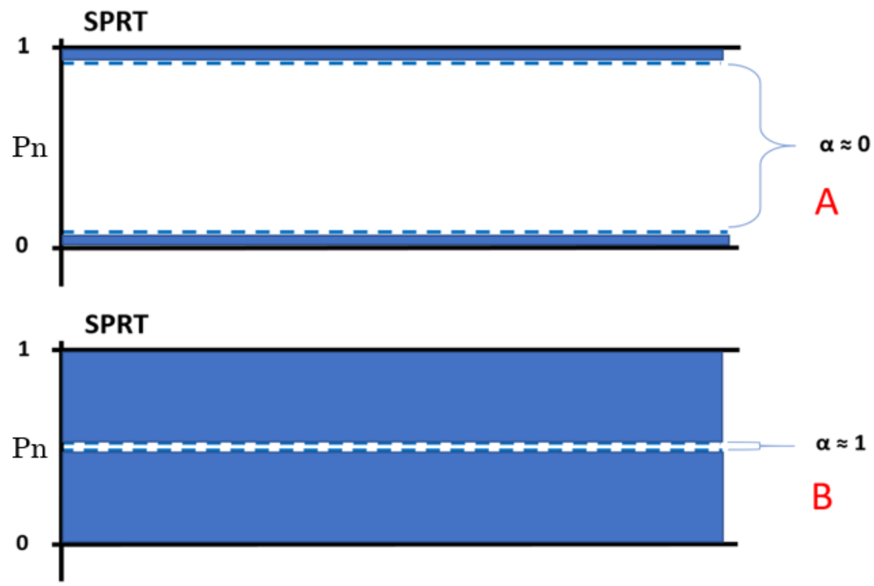

Figure 3.3: α value explanation, "α-Knob".

The white region, where samples continue to be collected, changes in size as α varies. The α component is also represented as a fraction, ranging from zero to one, of the total probability space. This region is centered about the starting probability value $P_n = 0.5$. It is easiest to think of this variability as changing the value of α by rotating a knob. For the purposes of this thesis, this will be referred to as α-Knob. This α-Knob region is bordered by a lower and upper stopping region, α and β respectively. The α and β values can both be located within the probability space. It was decided to keep the value of β fixed, when compared to α, for the purposes of this thesis. With β fixed, it was possible to choose values for α and keep the sample collection region centered about the initial P_n value.

The Sequential Analysis algorithm operates using a series of functions that use the initial conditions to analyze the data at each step. The required initial conditions are μ_0, μ_1, σ_0, σ_1, the probability for the state change to occur p, and the α-Knob value for the detection processes willingness to collect additional samples. After these values are determined, an initial measurement, Z_n, is calculated for use in the sampling process.

$$Z_n = \sigma N(\mu_n, \sigma_n^2) + \mu \tag{3.1}$$

Equation 3.1 is the measurement taken directly from the standard Normal distribution model, where $\mu_n = 0$, $\sigma_n = 1$, μ and σ are defined by the system state at the beginning of the test sequence. To accurately sample the Gaussian distribution of the system, it is important to normalize the data being collected at each stage as defined by the normalization process:

$$N(\mu, \sigma^2, Z_n) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1(Z_n - \mu)^2}{2\sigma^2}} \tag{3.2}$$

This normalization calculation is then conducted for both Gaussian Normal distributions as defined by the initial conditions. The results from Equation 3.2 are then used to calculate the probability density of the sample distribution, which corresponds to the sensor state at each stage as defined by Bayes's theorem below

$$P_n(Z_n) = \frac{P_n N(\mu_0, \sigma_0^2, Z_n)}{P_n N(\mu_1, \sigma_1^2, Z_n) + (1 - P_n) N(\mu_0, \sigma_0^2, Z_n)}. \tag{3.3}$$

Equation 3.3 is used to determine if a stopping threshold has been met. After each sample, the algorithm checks if one of the stopping thresholds were satisfied by referencing the probability density function to the stopping rules α and β, which were determined at the start of each test sequence. When $P_n < \alpha$ or $P_n > \beta$, the test sequence ends.

It was decided to use a colormap to symbolize the influence the Z_n value holds over the decision process calculated by Equation 3.3. The colormap chosen for use to assist in representing the results below was originally developed by [Krol, 2017] and symbolizes, through a three color (red, yellow, green) colormap, the overall distribution likelihood each Z_n sample yields during the testing process. Figure 3.4 graphically shows the association between Equations 3.1-3.3 and the developed colormap technique utilized throughout this thesis.

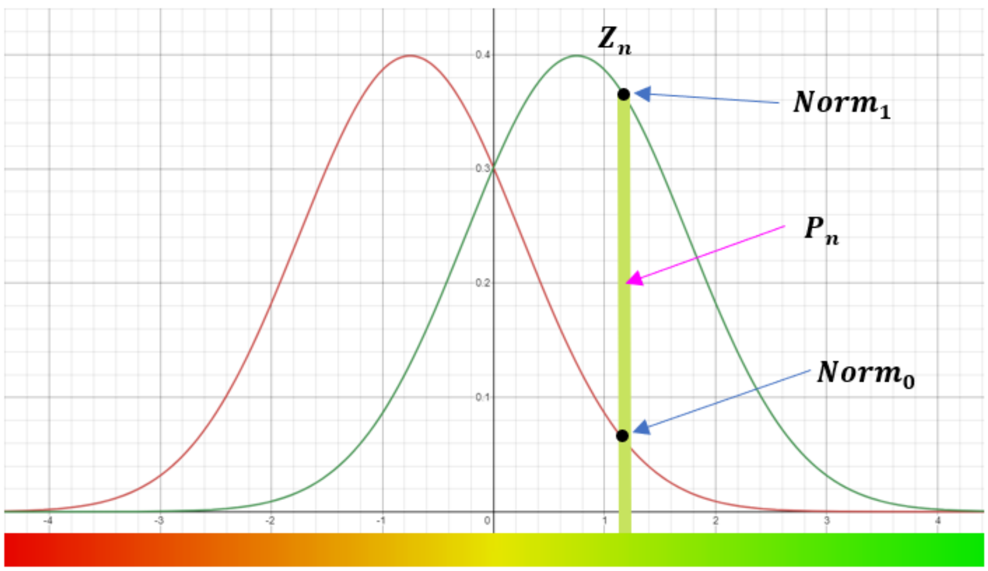

Figure 3.4: Graphical representation of associated interactions between Equations 3.1-3.3 and the developed likelihood colormap [Desmos, 2021].

Red signifies a more prominent influence from the μ_0 distribution introduced in Section 3.1. The region of the colormap represented in yellow are those measurements located within the regions shown in Figures 3.1 and 3.2 where the distributions overlap significantly and referred to as yielding high interference. That being the region centered about zero on the x-axis. Data measurements within this region are less likely to influence the decision model significantly due to the uncertainty attributed to these values. The green component of the colormap represents an influence imparted on the detection algorithm from the μ_1 distribution, also introduced in Section 3.1. With the colormap as a guide, assessment of the results from the single-shot case of the Sequential Analysis method can take place. The test results shown in Figure 3.5 were calculated using the *strong sensor* model.

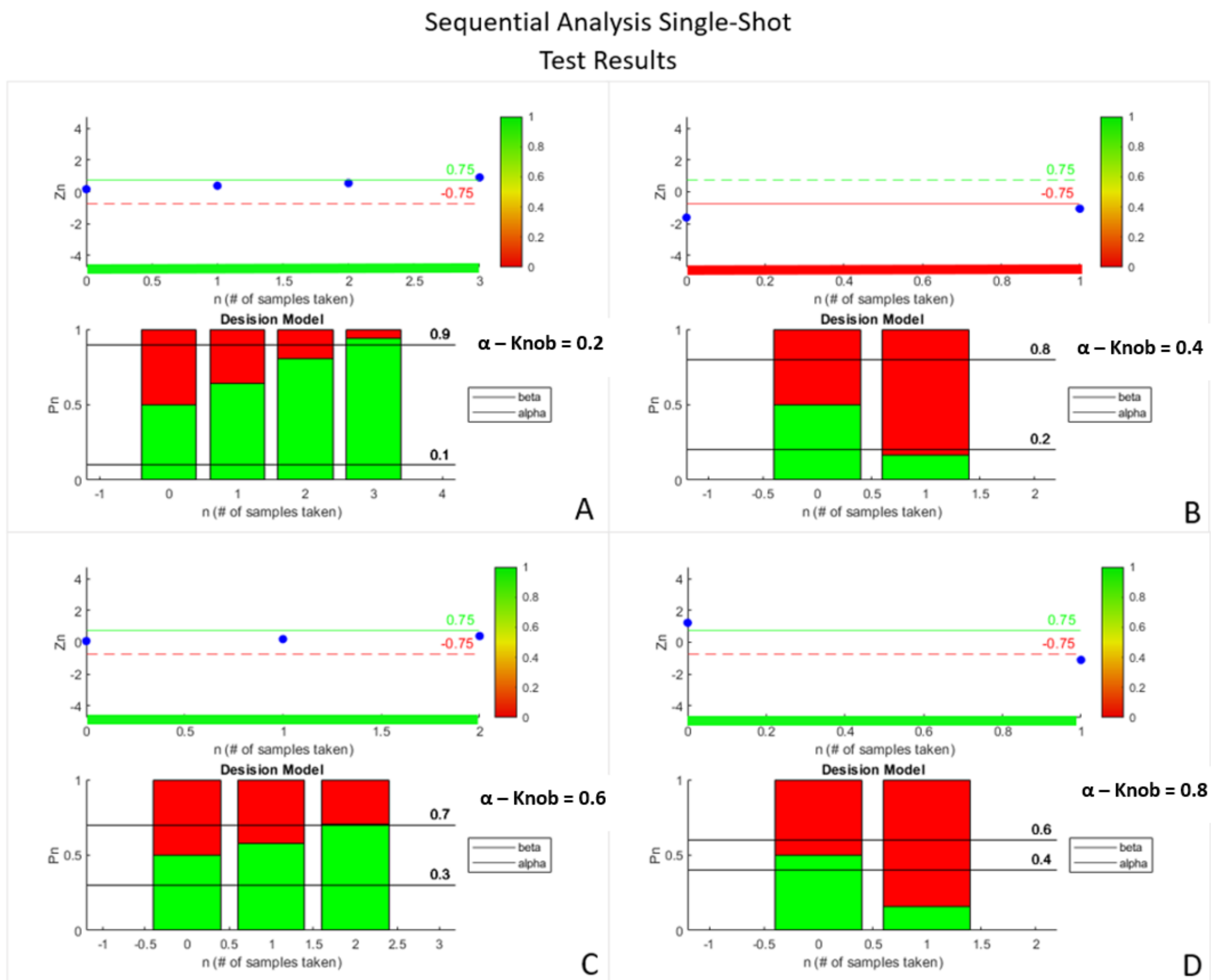

Figure 3.5: Test results from the *strong sensor* model.

These test sequences vary solely by changing the α-Knob value, which in turn adjusts the threshold values, $alpha$ and $beta$, from one test to the next. Figure 3.5A shows an α-Knob value of 0.2 and stopping threshold values 0.1 and 0.9 respectively, 3.5B represents an α-Knob set to 0.4 with thresholds 0.2 and 0.8, 3.5C utilizes α-Knob value 0.6 and thresholds 0.3 and 0.7, and 3.5D has an α-Knob value of 0.8 with thresholds 0.4 and 0.6. Figure 3.5A illustrates a test sequence where the true state was determined to be that of μ_1, shown by the green bar below the Z_n graph, and the solid green line on the Z_n graph, which symbolizes the true states mean value. Observing these results exhibits that the analysis, shown in the Z_n and P_n graphs, determined the $\mu_1 = 0.75$ distribution was present during this testing sequence. Furthermore, this can be seen by examining the incoming data samples shown in the Z_n graph, and by understanding the correlation each point has with the colormap shown to the right of the graph. These values occurred predominantly above the zero y-axis value and are more often attributed to the μ_1 distribution. If incoming samples would have occurred more frequently below zero, they would have been more often attributed to the μ_0 distribution. An example of this is shown in Figure 3.5B.

The detection algorithm determined the controlling distribution for this test sequence to be μ_0. As can be seen by the horizontal colored lines associated with the Z_n graph, the true state for this test was the μ_0 distribution. Therefore, the detection scheme analyzed and chose the correct distribution. Additionally, the algorithm collected two samples of data prior to making this determination. As Figure 3.3 demonstrated, when α-Knob approaches 1, the region associated with continuing to collect samples, the threshold gap, decreases in overall size. Continuing to increase the α-Knob value shows that the decision process gathers fewer samples and has a higher probability of making an incorrect distribution determination. Figure 3.5D represents an instance where the algorithm made an incorrect determination. This can be attributed to the smaller threshold gap afforded to the decision model, which in turn leads to fewer samples being collected, and the distribution overlap discussed previously. It is important to keep in mind the probabilities at which individually collected samples may occur when dealing with a Normal distribution. Figure 3.6 demonstrates these occurrence probabilities.

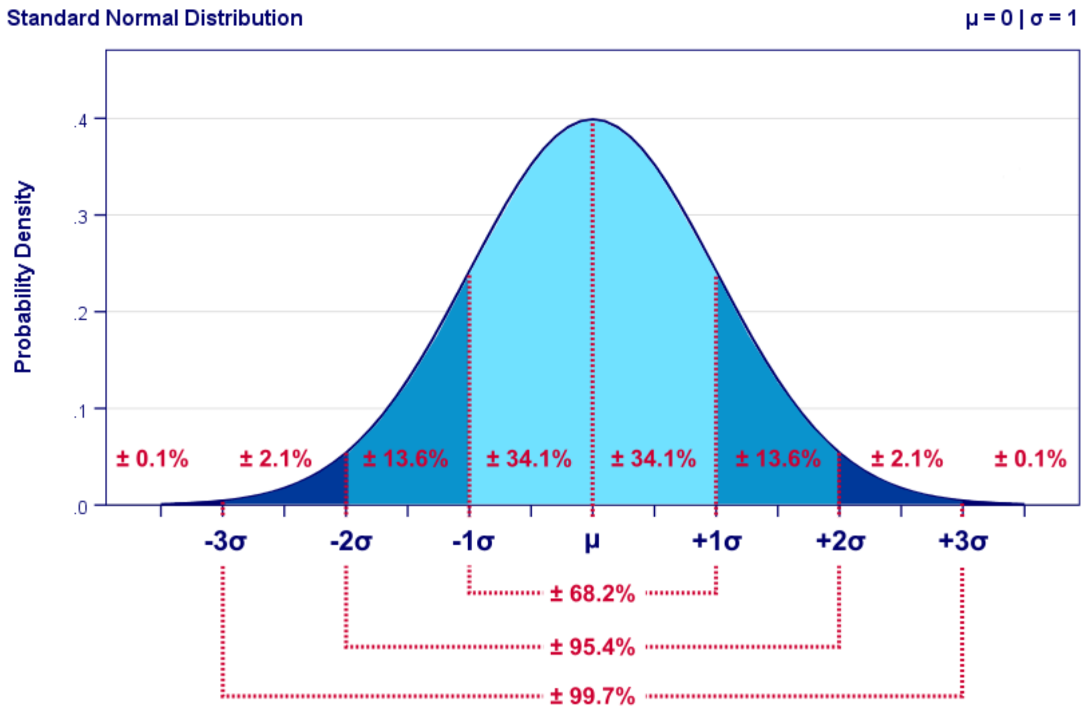

Figure 3.6: Standard Normal distribution probabilities of sample occurrence [SPSS, 2021].

Comparing the information illustrated in Figure 3.6 to Figures 3.1 and 3.2, it can be seen how the two distributions that make up the sensor model can make it difficult to yield correct decisions with absolute certainty, if these distributions overlap significantly. This is even more evident when examining Figure 3.6 in the region that makes up one standard deviation, σ, away from the mean value μ. Approximately 68.2% of all values will be sampled from this region. Cumulatively, 27.2% of additional samples will occur from the next standard deviation region. Figure 3.7 shows a series of results from testing performed with the *weak sensor* model described in Section 3.1.

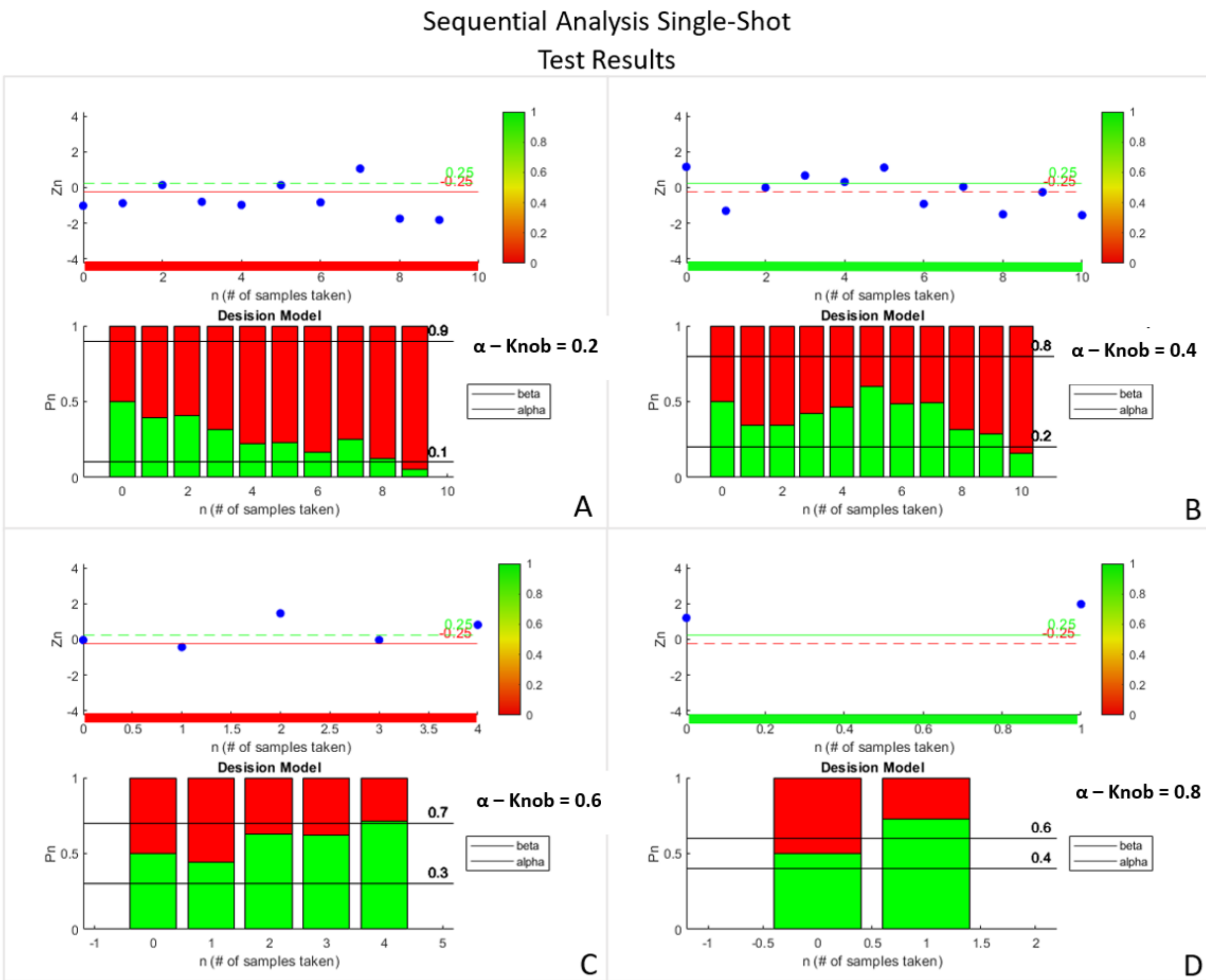

Figure 3.7: Test results from the *weak sensor* model.

Similar to the setup explained for Figure 3.5 above, the α-Knob value was varied to examine the resulting stopping threshold variability from the sensor model. An important note on the results that created graphs 3.7.B and 3.7.C, one can observe that the calculated solutions are incorrect. The detection model decided the controlling distributions were μ_0 and μ_1 respectively. As the true state data, represented on the Z_n graphs for each result, the true state distributions were in fact μ_1 and μ_0 respectively. This further lends credit to the idea of distribution overlapping interference discussed previously with Figure 3.6 and each of the sensor models, Figures 3.1 and 3.2. As illustrated through the use of Figure 3.6, the region comprising of 68.2% of all possible samples extends away from the mean value of a Normal distribution by one standard deviation. Since the *weak* *sensor* model consists of two Gaussian Normal distributions that yield an SNR of 0.5, they interact with each other significantly and allow for additional error to occur when making a decision on which state distribution is present at the time of the test sequence.

As can be seen in Figures 3.5 and 3.7, the number of samples taken from one test sequence to the next generally decreases as the α-Knob value increases and the stopping threshold gap decreases in size. As the gap between the stopping regions becomes smaller, resulting tests will utilize less collected samples when making a determination on which distribution is occurring during that sequence. Evidence of this can be observed in Section 3.2.2, where the error-vs-delay curves that are generated by progressively varying the α-Knob value and decreasing the gap between the two stopping thresholds, α and β, will occur until they are effectively equal to each other.

3.2.2 Multiple-Runs, Varying The α-Knob value

It is important to gain an understanding on how the Sequential Analysis method performs as the α-Knob value is varied. Mainly, it is desirable to see the correlation, if any, between delay and error as α-Knob is varied with a fixed sensor model. A process was developed that would initiate with α-Knob $= 0.01$, it would then run the Sequential Analysis test for 10,000 iterations, and average the decision values after each 10,000 iteration sequence. After completing these 10,000 iterations, the process would then increment the α-Knob value to the next position, α-Knob $= 0.02$, then it runs the 10,000 iteration sequence again. This process continues to increment until the range of α-Knob values is observed to be from zero to one, at which point the process compiles the results collected from each step into an α-Knob vs delay/error graph. Figure 3.8 shows the results for such a scheme when analyzing the *strong sensor*.

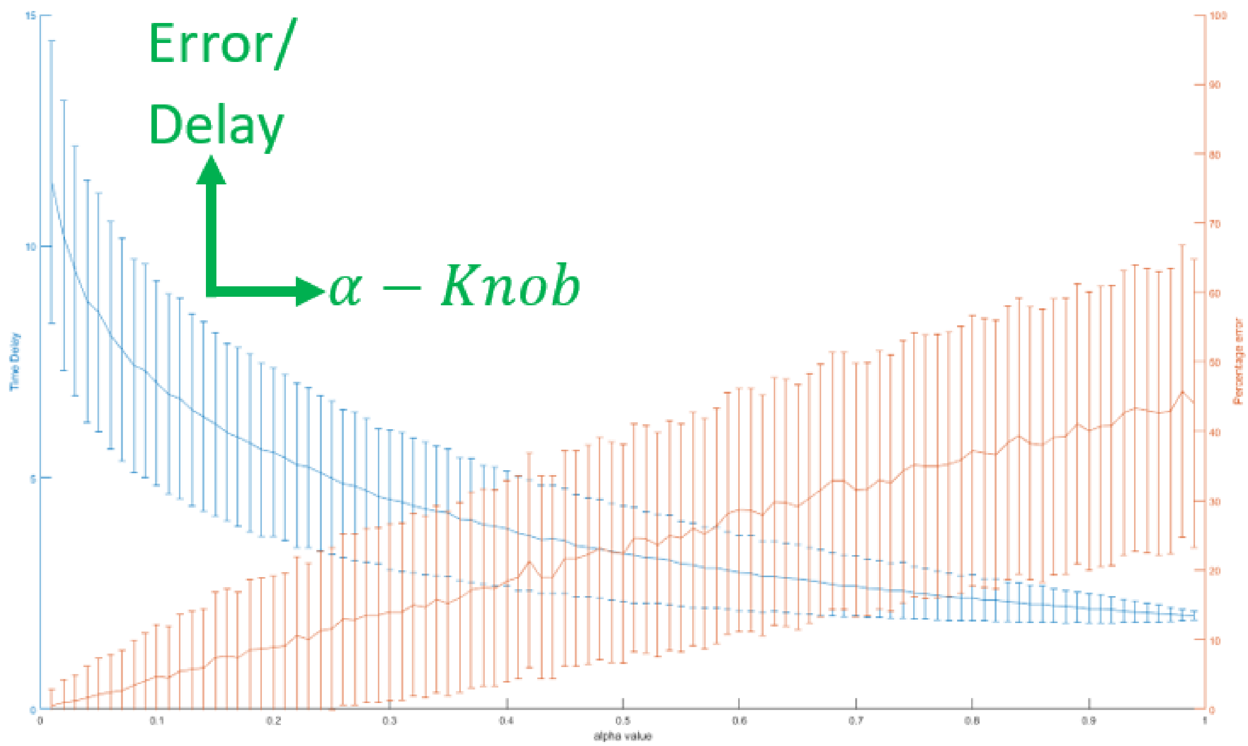

Figure 3.8: Varying α-Knob test results, *strong sensor*.

When α-Knob is close to zero, the highest amount of delay (shown as the blue line) and the lowest error rate occurrence (illustrated as the orange line) is observed. The low error rate is due to the algorithm being afforded the most amount of real estate between the test initiation value and the threshold gaps lower and upper values, thus allowing for more samples to be collected. As α-Knob approaches one, this stopping threshold gap decreases in width, i.e. α and β progress closer toward 0.5, the test sequence delay decreases and the associated error increases. In Figure 3.8 a crossover of both graphical components occurs approximately at α-Knob = 0.5. At this value, the delay and percentage error are minimized in relation to each other. This type of iterative analysis may be used for varying the initial conditions and threshold increments to determine a suitable set of upper and lower threshold values for any given test sequence with a known sensor model. Additionally, this method of analysis eliminates the need for manually progressing the threshold values to observe these changes.

After performing this analysis technique on the *strong sensor*, the process was replicated for the *weak sensor*. The results of this process can be seen in Figure 3.9.

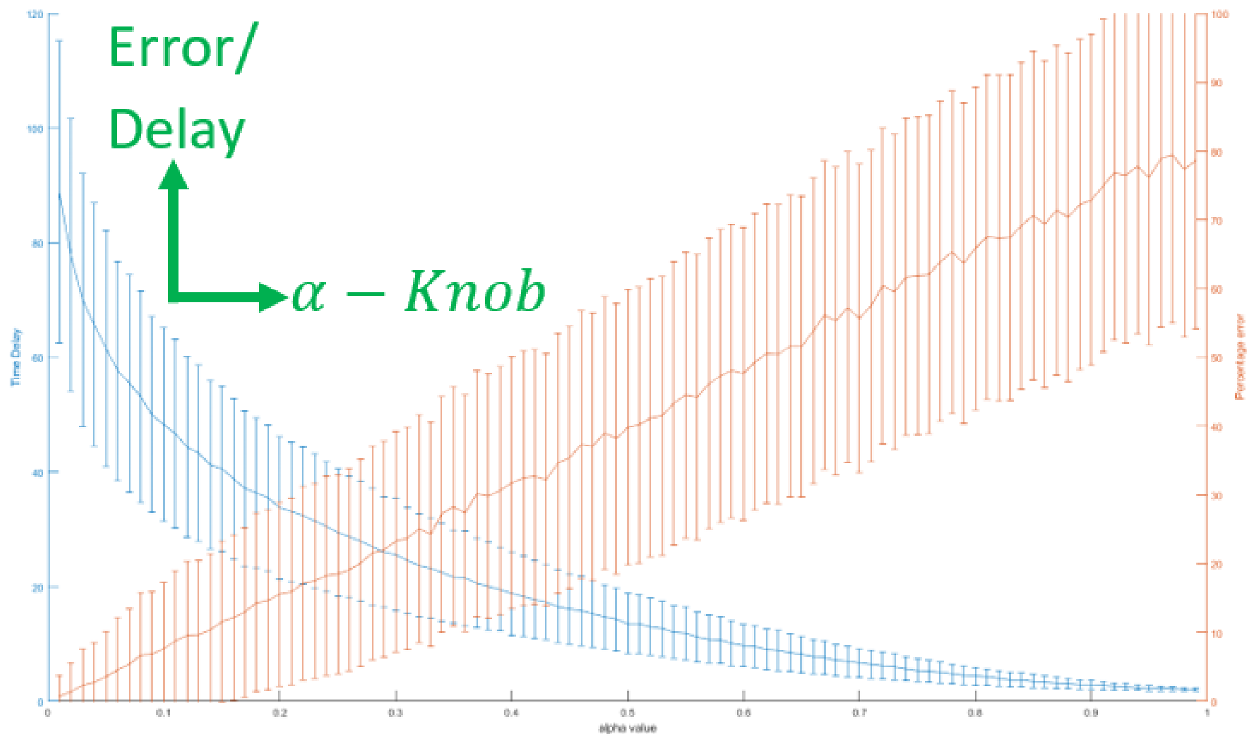

Figure 3.9: Varying α test results, *weak sensor*.

As expected, the time delay between the test initiation and the algorithm deciding which distribution it found was being sampled saw a total increase. Additionally, an increase in the total error amount can be observed. Both of which can be explained similarly to the incorrect decisions made in Figure 3.7, the Single-Shot figures found in section 3.2.1. This is due to the distributions that make up the *weak sensor* model being closely spaced and therefore yielding a large amount of interference in the regions where the majority of sampling would occur. It was expected to see an increase in delay since the algorithm is willing to sample more often. Additionally, it was expected to see more error since the sensor model normal distributions were influencing each other to a larger extent when compared to the *strong sensor*.

3.3 Quickest Detection Algorithm

Chapter two, Section four illustrated that the Quickest Detection statistical analysis method (also referred to as a Quickest Probability Ratio Test (QPRT)) is an adapted form of Wald's Sequential Analysis technique, SPRT. The QPRT algorithm is designed to begin operation with the prior distribution being defined as a functional system. It is only after an unknown amount of time that the distribution for the test sequence changes to a non-functioning distribution. Mentioned previously, the Quickest Detection method is tasked with detecting the occurrence of a change-point and making a stopping decision based on the pre-determined stopping threshold value.

In terms of the Sequential Analysis method, α is defined as a region of probability where a testing sequence continues to gather data. As the value of $\alpha \Rightarrow 0$, the testing sequence will continue everywhere, and when $\alpha \Rightarrow 1$, the sequence will stop everywhere. Figure 3.3 was used to illustrate this in the previous section.

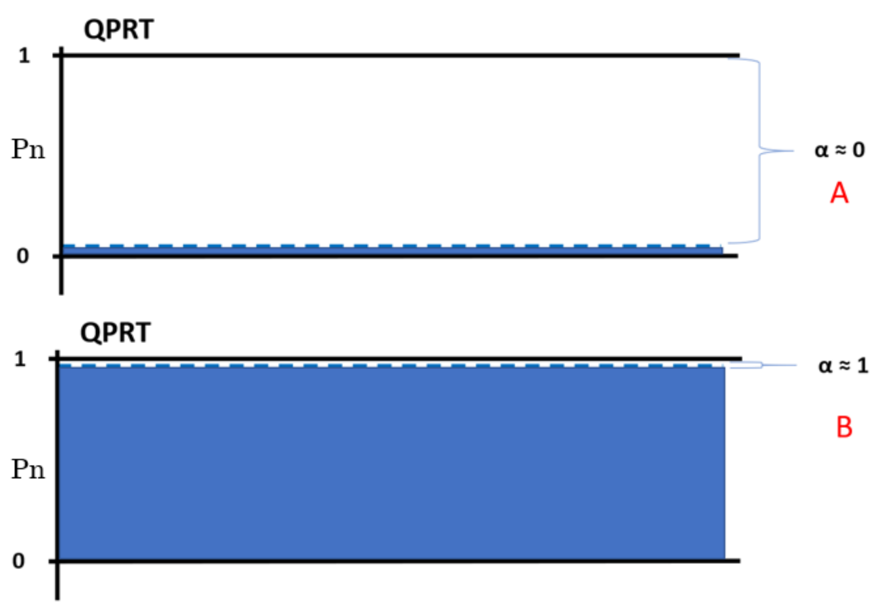

Figure 3.10: α value explanation, "α-Knob".

As Figure 3.10 shows, the white region, where samples continue to be collected, changes in size as α varies. The α component is also represented as a fraction of the total probability space, ranging from zero to one. Similar to the understanding when discussing Sequential Analysis, one can think of this variability as changing the value of α by rotating a knob, and this thesis will continue to refer to this aspect as α-Knob. As one can see, this region is bordered only by a lower threshold, described from here on as α. A key difference between this statistical method and the Sequential Analysis method is that Quickest Detection only utilizes one bounding threshold. Therefore, the α-Knob value and α represent the same component when discussing Quickest Detection. Keeping this in mind, when discussing Quickest Detection α-Knob will be referred to as the stopping threshold α.

3.3.1 Single-Shot Quickest Detection

Similar to the Sequential Analysis method, Quickest Detection utilizes user specified initial conditions of the sensor model. Figure 3.11 illustrates the operation of a single-shot detection process using the Quickest Detection method. At the beginning of the testing sequence, when performing a Quickest Detection statistical process, the initial system state is known. In this problem formulation, the system is in a functional state, μ_1 and after a random, unknown, time interval the system will undergo a state change to the non-functional state, μ_0. The goals of the Quickest Detection method are to optimize two performance indices: the delay between the time a change occurs and when it is detected, and the measure of the frequency of false alarms [Poor and Hadjiliadis, 2009]

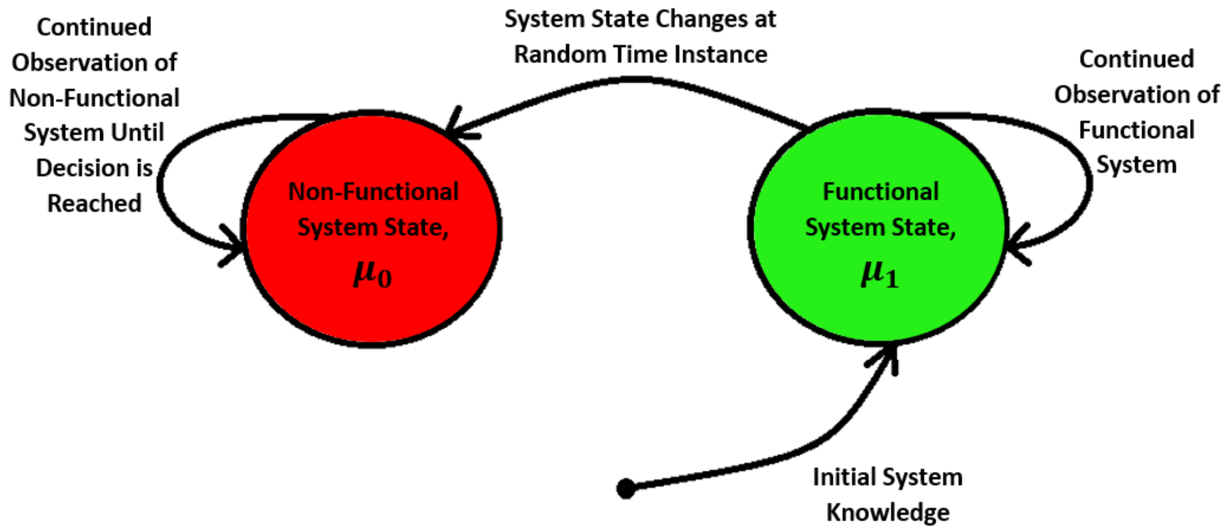

Figure 3.11: Example test sequence explaining operation of Quickest Detection problem formulation.

After the prior stage measurement, the sensor model is known to the algorithm but not to the testing sequence. This information is stored for later use. A per-stage measurement is then taken and analyzed to determine if the stopping threshold α is achieved. If the stopping threshold is not satisfied, another measurement is taken. Once the stopping threshold is satisfied, the algorithm defines the distribution of the test sequence. After the distribution is defined, the total cost of the test is calculated via the type error which occurred during the test, as well as the cumulative per-stage cost. The type error is defined as no error, type I error (false positive), or type II error (false negative). Each of these three solutions yield a different impact over the total cost of the test as defined by the overall testing cost function. The per stage cost is calculated by the total number of times data was collected and multiplied by the cost attributed to continuing the testing sequence.

There are a few key differences between the Sequential Analysis and Quickest Detection techniques. When using Sequential Analysis, the system state distribution is the same throughout the test and is unknown. In the Quickest Detection scheme, a system state change occurs at a random point during the testing sequence. In Quickest Detection algorithms, this change-point is determined by a geometric distribution [Poor and Hadjiliadis, 2009], illustrated in Equation 3.4.

$$f(x) = k(x) = p(1-p)^{x-1}; \tag{3.4}$$

Equation 3.4 will be defined as k, where k is used to calculate the true time instance when the system state change will occur. This k value is not known by the testing algorithm until the end of the test when the delay is calculated. The delay can be calculated similar to the Sequential Analysis method, but will not attribute higher costs for error type occurrence at this time. This is due to the overall function of the Quickest Detection algorithm. As illustrated later in this section, the occurrence of detecting the change-point at the exact time it takes place is statistically unlikely. To this point, observations are merely for the occurrence of an early detection, known as a type II error (false negatives), or a delayed detection, a type I error (false positive). As further discussion in the next section will illustrate, multiple test runs and varying the α decision region will be used to investigation and understand the role this α value plays in the occurrence of stopping early and stopping late scenarios.

Like the Sequential Analysis algorithm previously developed, the Quickest Detection algorithm operates through the use of a series of functions that utilize predetermined initial conditions to analyze the data at each time instance. These functions are the same as those presented in Section 3.2 of this thesis. The required initial conditions are μ_0, μ_1, σ_0, σ_1, the probability of occurrence for the state change p, and α-Knob; the value defining the stopping threshold α. After these values are determined, an initial measurement, defined as Z_n in Equation 3.1, is collected for use in the sampling and decision process. Equation 3.1 is used for taking a direct measurement from the standard normal distribution model, where $\mu = 0$ and $\sigma = 1$. Similar to the Sequential Analysis case, in order to accurately sample the Gaussian Normal distribution of the system, the sample data must be normalized at each stage of collection as defined by the normalization process in Equation 3.2. The normalization calculation for both of the possible Gaussian Normal distributions that make up the sensor model is conducted. The results from Equation 3.2 are then used to calculate the probability likelihood ratio of the sample distribution for the sensor model, corresponding to the sensor state as defined by Bayes's theorem in Equation 3.3. Like Sequential Analysis, Equation 3.3 is used in the Quickest Detection method to determine if the stopping threshold, α, has been achieved. After each sample, the algorithm determines if this threshold is satisfied by referencing the probability density function to the α threshold. If α is satisfied by the logical test, $\alpha > P_n$, the test sequence ends. Figure 3.12 shows results gathered during a series of tests from the developed Quickest Detection algorithm when it was observing the *strong sensor*.

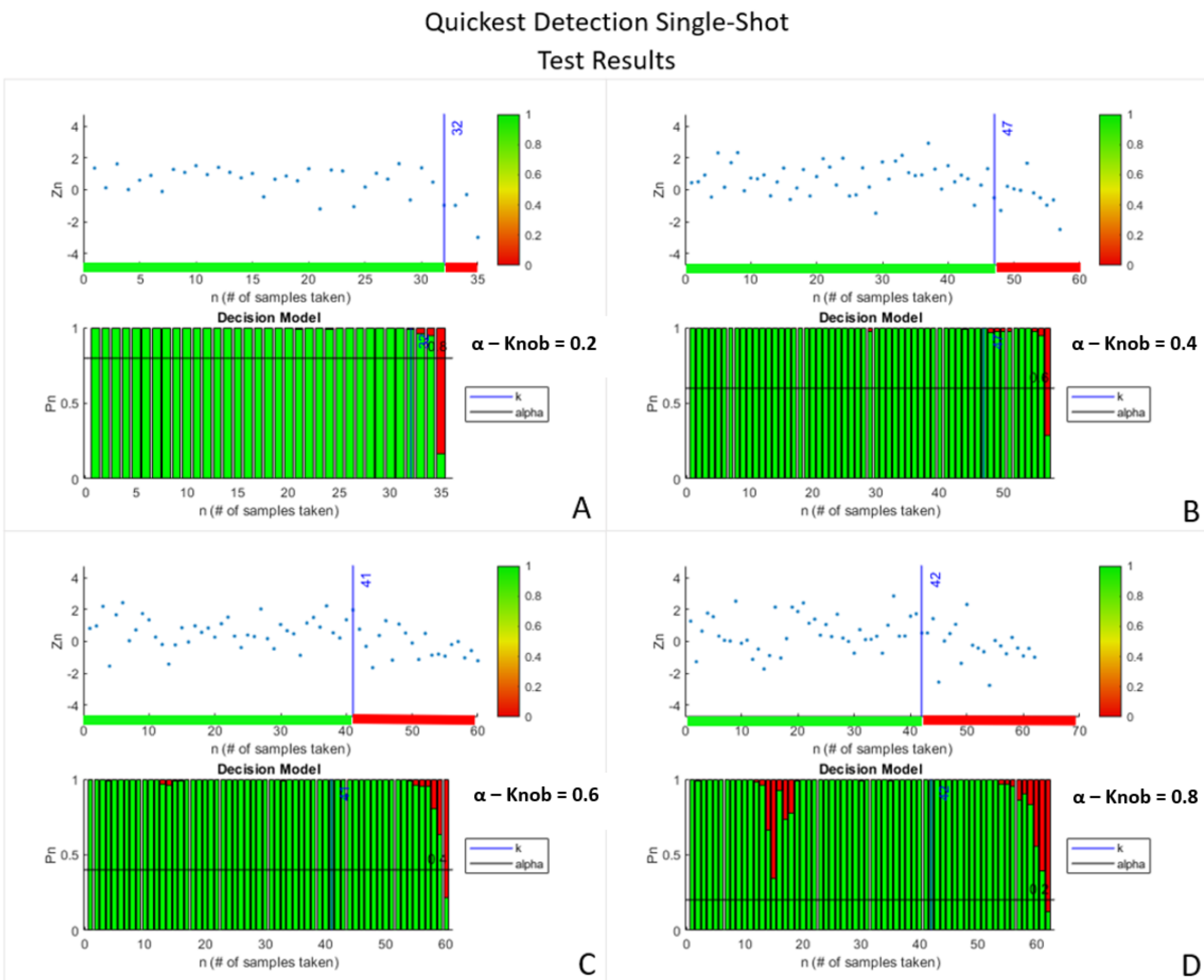

Figure 3.12: Test results from the *strong sensor* model.

Figure 3.12A represents results when the threshold, α, was set to 0.2, 3.12B shows a threshold of 0.4, 3.12C used a threshold of 0.6, and 3.12D used 0.8. Results are displayed similar to those discussed when analyzing results from the Sequential Analysis algorithm. There are several differences between the two. First, the colored bar below the Z_n graph is now being represented as two segments, green and red. Additionally, the true state indicators have been changed to a change-point time indicator. Green-colored segments of the graph indicate the true state of the system as being functional, since in Quickest Detection, the true state is defined as functional at test initiation. Then, at a random point in time during the testing sequence, the system state changes to the non-functional distribution, μ_0, shown as the red bar below the Z_n graph. Additionally, the Z_n graph shows the true state change-point k through the use of the vertical blue line. The overall detection method is similar to that of Sequential Analysis, in that when the measurements being collected significantly influence the decision model, the algorithm determines the system change has occurred and ends the testing sequence.

Figure 3.12A shows results gathered from the *strong sensor* with an α value of 0.2. A visible difference in the measurements on the Z_n graph can be observed when the change-point occurred. A negative shift in the sample mean can be observed at that change-point, further indicated by the colormap correspondence. After the change-point occurred, a majority of the samples trended toward the μ_0 distribution. When comparing Figure 3.12A to the other graph sets, this physical data change is less apparent, but one can still see that the change-point occurred and the detection algorithm alerted to its occurrence. The delay prior to this alert increases as the value of α increases. This increase leads to additional samples being collected, which in turn leads to the algorithm being more certain that a change-point occurred. Additionally, when comparing graph set Figure 3.12A to the other graph sets, there is a significantly shorter delay time from when the change-point occurred to when the algorithm detected it.

After observing the results from the *strong sensor*, the same series of tests using the *weak sensor* were run. Identical setups and stopping thresholds were used for both tests. Figure 3.13 illustrates the results from these tests. As illustrated, the length of time delay after the change-point occurrence is significantly longer. This is not surprising, when comparing these results to those from the analysis of the *weak sensor* in the Sequential Analysis algorithm in Section 3.2.

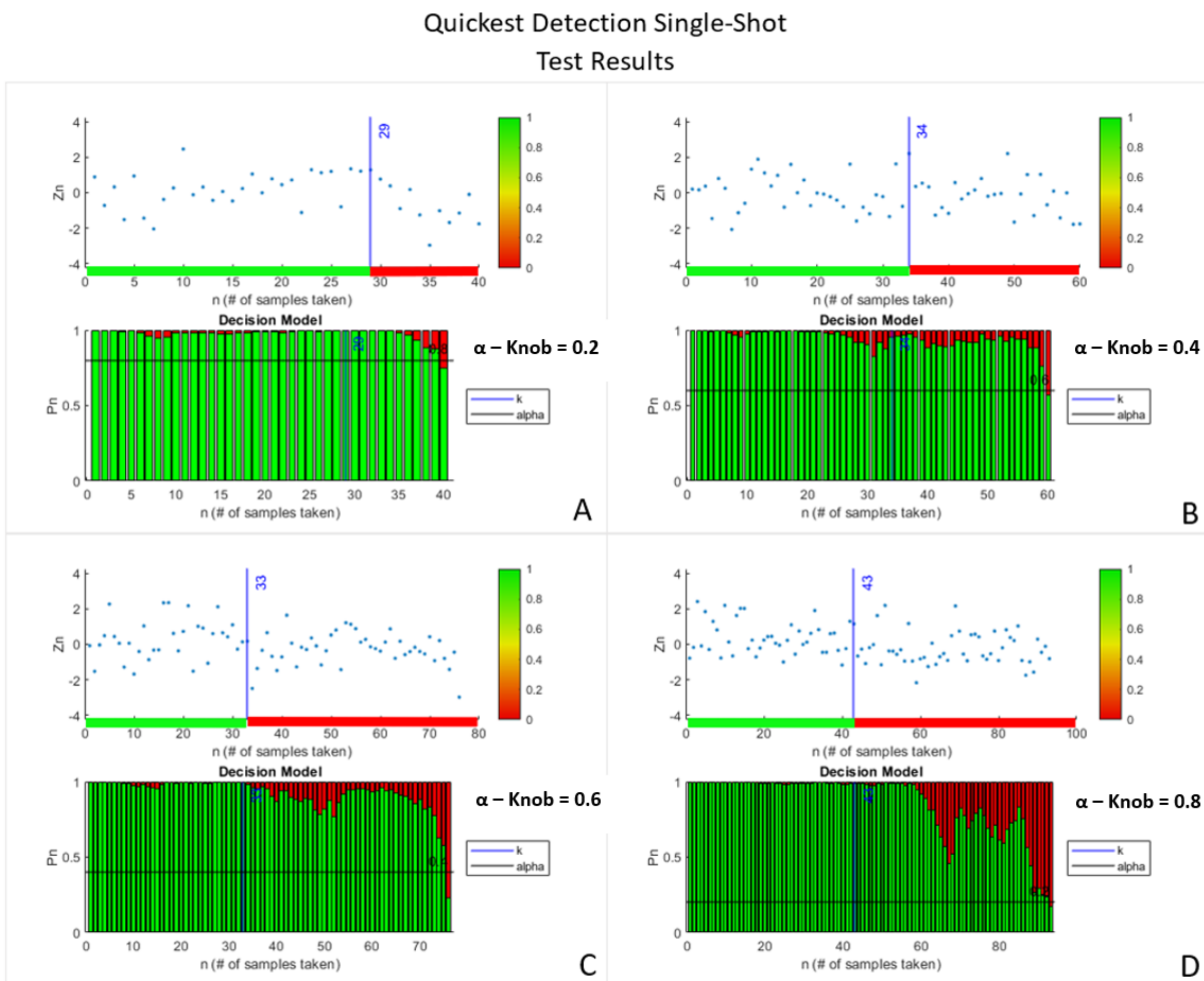

Figure 3.13: Test results from the *weak sensor* model.

Each of the graphs shown in Figure 3.13 resemble results where the α-Knob value was varied to show results similar to those in Figure 3.12 when observing the *strong sensor*. Figure 3.13A illustrates results when α-Knob was set to 0.2. The change-point can be observed at time instance 29 via the vertical blue line. The algorithm continues to collect samples for another eleven time instances before making its stopping decision. Similar results are shown in Figures 3.12B-D. The algorithm experiences a state change after the change-point occurs and makes the decision to stop within an appropriate amount of time for the weak sensor model being tested. The results shown in Figure 3.13 did not produce results where the decision model stopped testing early. This would have been shown by a figure not having the vertical blue line, signifying the occurrence of the change-point during the test. Furthermore, such a result would illustrate a type II error. Investigation through multiple-runs, and varying the α-Knob value in the next section will be used to determine values of α-Knob that produce false alarms of type I and type II errors. By varying the α-Knob value the stopping decision threshold may allow for the occurrence of additional type II error decisions (false-negatives) for each of the sensor models.

3.3.2 Multiple-Runs, Varying The α-Knob Value

When examining the Sequential Analysis case using the method where α-Knob was varied, a relationship between the delay and error rates was observed. Subsequently, when varying the α-Knob value, the α and β thresholds were varied which decreased the threshold gap. Results similar to these were less useful when examining the Quickest Detection method. This was due to the fact that, statistically, the algorithm would not alert of a state change at the exact moment it occurs. By not alerting a state change at the exact moment of occurrence, the algorithm introduced a new, potentially misleading variable into the analysis. This variable was originally defined as the error associated with the α-Knob value being tested. Instead it was found to more useful to examine the rates at which stopping early and stopping late occurred at each value of α-Knob. In order to test the frequency of early and late stops, the same series of 10,000 iterations was run using an initial stopping value of $\alpha = 0.01$. After these iterations were completed, the average delay value was determined, and the occurrence of early and late stops was catalogued. Next, the threshold was progressed to $\alpha = 0.02$ and the process was repeated until the threshold value was $\alpha = 0.99$. At this point all of the values were saved and shown in graphical form as Figures 3.14 and 3.15 illustrate.

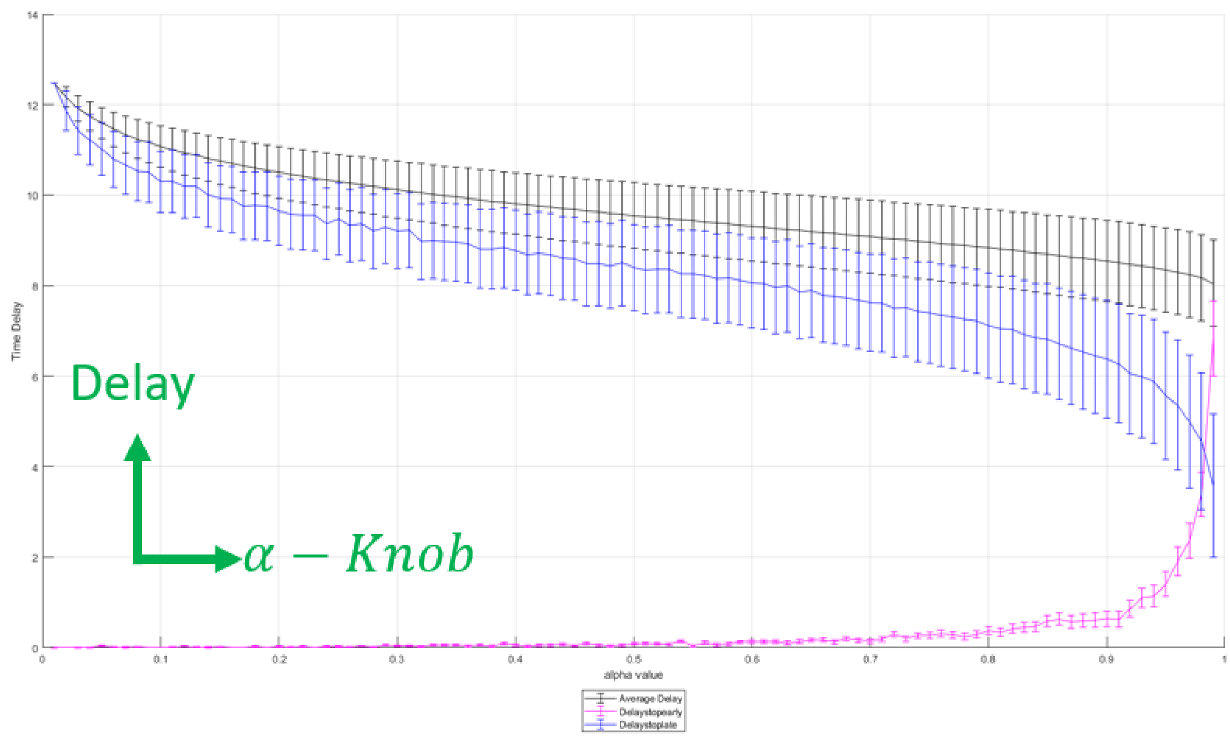

Figure 3.14: Varying α test results, *strong sensor*.

Figure 3.14 represents these results when investigating the *strong sensor*. The average delay, denoted with black dots, decreases over time as the α value draws closer to one, which allows for faster alerts of a state change to occur. Additionally, the rate of stopping early increases exponentially as the threshold approaches one, denoted by the magenta line. This denotes an increased occurrence of false negatives (type II errors).

Now, viewing the same form of analysis on the *weak sensor*, shown in Figure 3.15, similar trends can be observed. The delay at the lower values of α was observed to be higher than what can be seen as α approaches one on the x-axis. This is shown by the black and blue lines having higher values of delay on the left side of the graph than they do on the right side of the graph. Conversely, a similar trend can be seen occurring with the stop late results denoted by the magenta line as was observed in Figure 3.14. There were less stop late results at lower values of α than there were at upper values.

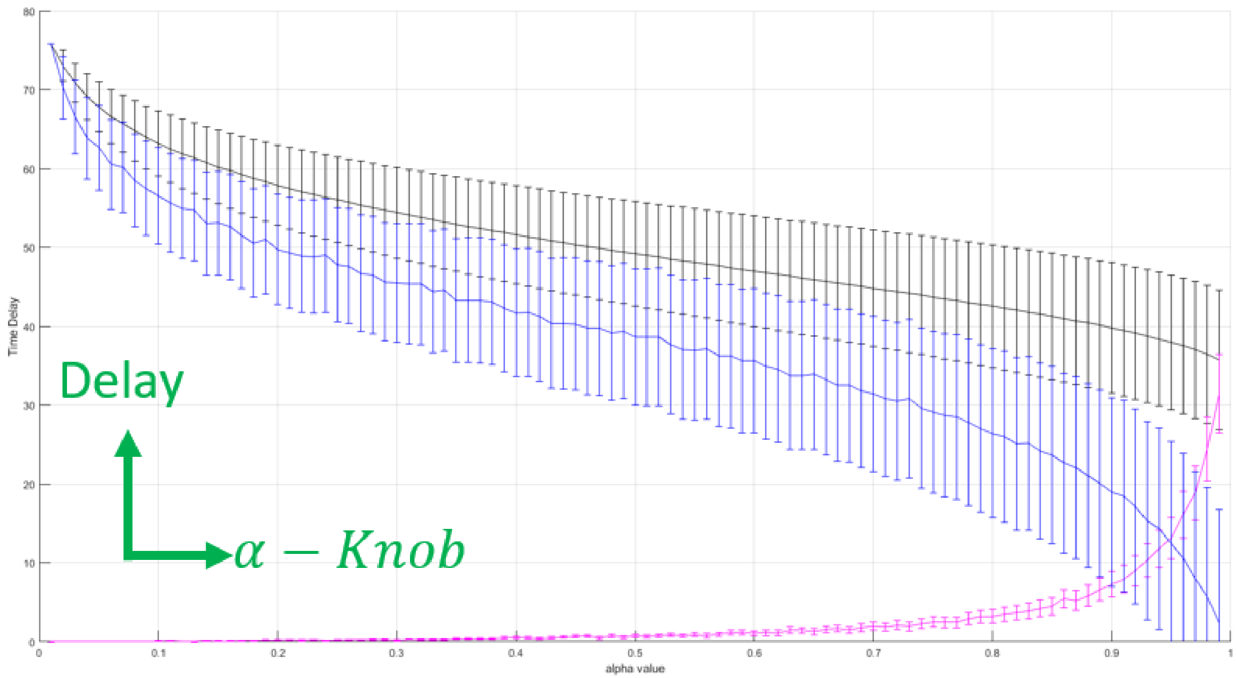

Figure 3.15: Varying test results, *weak sensor*.

The main difference in Figure 3.15, when compared to Figure 3.14, is shown by a more gradual increase in the early stop curve, denoted in magenta. The late-stop curve, denoted in blue, resembles a sharper decline at the end of its range when analyzing the *weak sensor* than was exhibited in the *strong sensor* model. This more gentle trend in the magenta line presents more stop early events at lower values of α then those shown in Figure 3.14. From Figures 3.14 and 3.15, one may begin to gain insight into the expected results of further single-shot test sequences.

3.4 Comparative Analysis

Through this thesis and the analysis prior, a number of similarities and dissimilarities between the statistical methods of Sequential Analysis and Quickest Detection were found. Both methods function to determine the state of the system being tested at reasonable speed by minimizing delay and possible false alarms. Both utilize continually collected distribution data to form decisions based on the utilization of the likelihood ratio technique. Each of these methods handle sensor models developed with the use of various Gaussian Normal distributions, and can be adapted for the use with other sensor models. For the purposes of this thesis, exploration into the use of these analysis methods when analyzing additional or dissimilar distributions other than those developed in Section 3.1 was not conducted. Research shows that these techniques possess capabilities to be manipulated to perform the same techniques of detection with various sensor model types [Poor and Hadjiliadis, 2009] [Aminikhanghahi and Cook, 2017][Lai, 2001].

Differences between the two methods include the manner in which α-Knob is used during the testing process, the scheme of use each method affords the test initiator, the operative function controlling the state determination in each, and the various results readily available when performing in-depth analysis. The Sequential Analysis method operates through the use of α-Knob, bounded by a lower and an upper threshold α and β, respectively. It is through these thresholds that the algorithm determines when to stop collecting samples and make a determination on the system state. The Quickest Detection method also uses α-Knob, but it is bounded by only the lower threshold, α, which is also used to terminate the testing sequence. Additionally, the Quickest Detection method begins with a known initial state and continues collecting samples until results indicate a change of state has occurred. The α sets the level of the decision region threshold and the probability likelihood crosses this threshold so that the testing sequence concludes.

Each of these methods are useful in different ways when developing product failure mitigation schemes. The Sequential Analysis method is useful when the state of the system being tested is unknown. Conversely, the Quickest Detection method requires that the initial state of the system be known so it may develop an understanding of the system's "normal" circumstances of operation. Because of this, it is more accurate when determining if the state has changed. There are additional dissimilarities between the ways the system state determination is controlled. For the Sequential Analysis method, the state is determined at the beginning of the test, kept secret from the testing process, and used after a determination has been made to calculate any potential error type occurrence. The Quickest Detection method starts the test sequence with an initial state as functional, and through the use of the geometric distribution determined change-point, the state changes after an unknown amount of time to the non-functional state. Through discrete-time series testing, the algorithm monitors the collected system measurements until the α threshold value is satisfied and makes a determination that a state change has occurred. Lastly, a difference in the useful results provided through the in-depth multiple-runs analysis was observed. Through this method, the Sequential Analysis technique clearly demonstrates the delay and error rates as the α-Knob value is varied. When investigating the Quickest Detection method through similar means, useful results are gathered and displayed by a comparison of the delay and test termination type, that being an early or late stop.

Chapter 4

Recommendations and Conclusions

At the completion of this research, there remains additional work to pursue as these detection techniques continue to be developed. These include algorithmic improvements to better suit the detection/decision models currently being used in this thesis, and a combination scheme of the algorithmic techniques into a multi-level detection tool. In this section, the exploration of remaining topics which were either originally believed to be within the scope of this thesis, or through the analysis process, found to potentially enhance the future performance of the detection processes developed will be presented. These topics were determined to be better suited for investigation at a later date as research in the field continues to progress.

4.1 Trend Analysis

Trend Analysis is a technique believed to potentially assist in minimizing the number of samples to be collected and further serve to mitigate the occurrence of false alarms experienced during testing sequences. Trend Analysis is a process that estimates future changes through the analysis of previously collected data [Shilpy Sharma and Obimbo, 2016]. There are examples of research similar to this thesis that utilize this technique to shorten the total delay observed specifically in SPRT and QPRT decision models. As [Shilpy Sharma and Obimbo, 2016] illustrate, the most widely used techniques for Trend Analysis are Mann-Kendall, Spearman Rho, seasonal Kendall, and Cox-Stuart tests. The Mann-Kendall method is a non-parametric test that works for all distribution types, and is used to analyze collected data for increasing and decreasing trends that occur over time [Glen, 2016a]. The Spearman Rho technique is a non-parametric rank correlation coefficient used to measure the strength of a data-sets monotonic relationship, where if one data attribute changes, other attributes change similarly [Glen, 2016c]. Seasonal Kendall is another non-parametric technique used when performing monotonic data analysis on recorded samples influenced by seasons [Glen, 2016b]. Finally, the Cox-Stuart analysis method determines if collected data, assumed to be independent observations, are actually dependent on a trend occurring over time [Heckert, 2015]. Further study into the implementation of one or more of these concepts would strengthen the decision/detection models of the current techniques illustrated in this thesis. An example where Trend Analysis may have assisted in making a stopping decision sooner is shown in Figure 4.1.

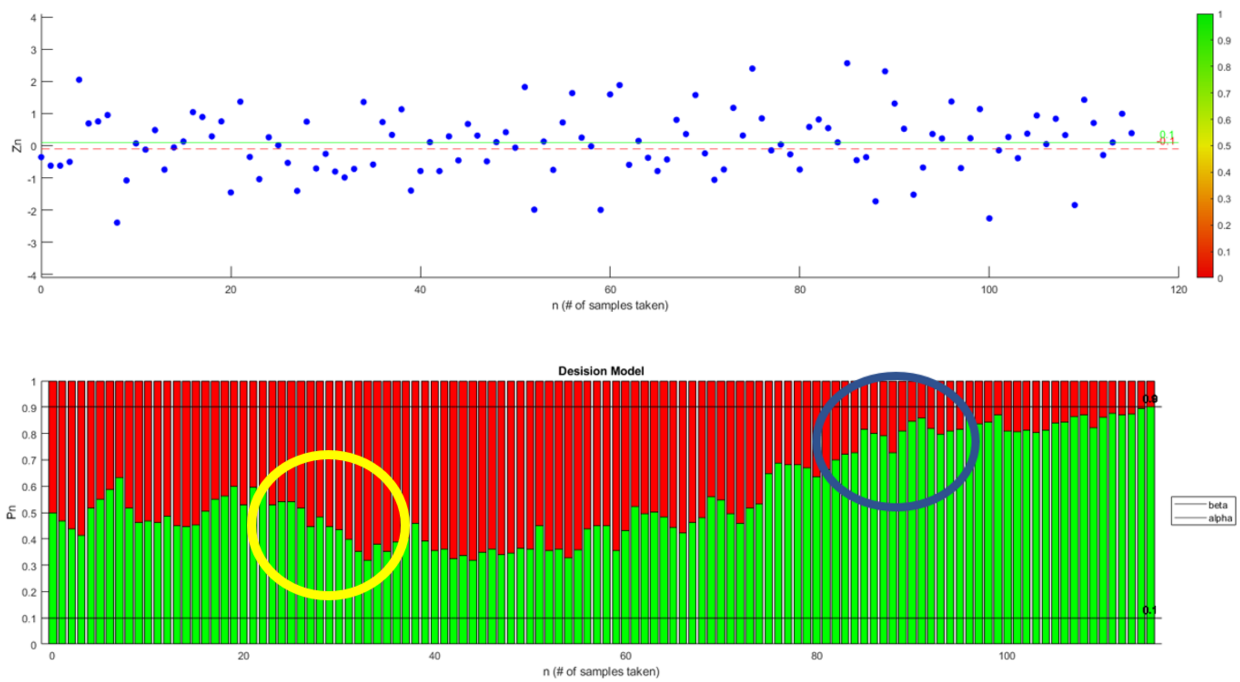

Figure 4.1: Trend Analysis techniques could lead to a decrease in decision process delay.

Figure 4.1 illustrates that, due to the probability likelihood not having yet surpassed the upper β bounding threshold, the test sequence was forced to continue until the likelihood of failure became large enough for a decision to be made. The use of a Trend Analysis technique may potentially decrease the overall delay time when deciding the system state for poor sensor models. Figure 4.1 is a representation of a poor sensor model using a dual Gaussian Normal distribution setup, which possesses an SNR of 0.3, and both distributions having standard deviation, $\sigma = 1$. There is a gradual increase in probability likelihood, beginning around time instance $n = 60$, which begins to taper off around time instance $n = 85$, circled in blue. The system state decision was further delayed until approximately time, $n = 115$. Through the use of Trend Analysis techniques, the system state could have potentially been determined quicker by analyzing the probability likelihood trend as it increased over time during the test sequence.

It is important to note, that the use of Trend Analysis can also harm detection accuracy by making inappropriate determinations based on the incoming data. This may lead to misinterpreted trends that lead to the occurrence of false alarms in the detection process. As Figure 4.1 shows, the yellow circled data resembles a negative trend in the collected sample probability likelihood. Through the use of Trend Analysis, trends similar to these possess the potential to be misinterpreted and could lead to false detections decisions. Specifically, in the case shown in Figure 4.1, a type II error would have occurred. Depending on the circumstance being observed, whether it be a machine used in manufacturing or similar, such false alarm detections may lead to significant delays in production and associated costs. Trend Analysis is a potential tool that may prove to be useful for implementation into the techniques developed in this thesis, but its usefulness and accuracy must be determined through in-depth analysis prior to execution.

4.2 Optimal Stopping

Optimal Stopping theory is a technique based on Wald's SPRT method [Siegmund, 2003]. The use of Optimal Stopping is to minimize any expected cost or maximize rewards [Zhongxiang Dai and Jaillet, 2019]. This formulation which analyzes for a suitable Optimal Stopping decision region, when associated to the problem of product functionality discussed throughout this thesis, would deal with negative rewards (losses) [Siegmund, 2003]. This is due to the SPRT and QPRT techniques means of analyzing and making decisions in the form of losses (costs). When performing Optimal Stopping, a decision is made to either stop sampling and pay the current cost or continue sampling in the belief of paying a lower cost by making a more accurate decision at any time n [Siegmund, 2003]. This is done similarly in the formulations for SPRT and QPRT discussed previously, which use a stop-and-decide model based on varying the α-Knob value [Bai and Gupta, 2017].

The use of Optimal Stopping in later iterations of this developmental research would allow for the α-Knob value and subsequent stopping thresholds to be calculated at the time of each testing sequence initiation. This operation is understood to minimize the amount of interaction from the researcher/test initiator and allow the algorithm to run autonomously without human interference. For the implementation of such a scheme, the sensor model would be input into the respective optimization algorithm, at which point the optimal α-Knob value for the specific sensor model would be calculated. After the α-Knob value is configured, the associated stopping thresholds would be implemented into the overall test sequence algorithm for utilization.

4.3 Composite Detection Algorithm

A combination decision/detection process, using a multi-level detection scheme is an additionally proposed next step for the progression of failure recognition techniques. The belief that the importance of such work is not only unique, but a necessary addition to current mitigation and early warning processes remains an important next step in the field of failure recognition. [Shilpy Sharma and Obimbo, 2016] states that very little research has been conducted at the time of their survey that showed the utilization of combination testing methods such as this proposed technique. Furthermore, [Shilpy Sharma and Obimbo, 2016] states that typically only one detection method is used at a time in research settings, such as Change-Point Analysis, Hypothesis Testing, Optimal Stopping theory, or Trend Analysis. They presented the idea that only utilizing singular analysis methods could allow for data masking to occur, thereby decreasing the accuracy of prediction models. The use of such a multi-level observational techniques could assist in discovering masked fault occurrences and enhance the accuracy of decision making and detection techniques currently in use for failure recognition. The deployment of a composite detection scheme would, ideally, include the use of multiple-level observation techniques. Figure 4.2 illustrates one possible scheme that was under consideration.

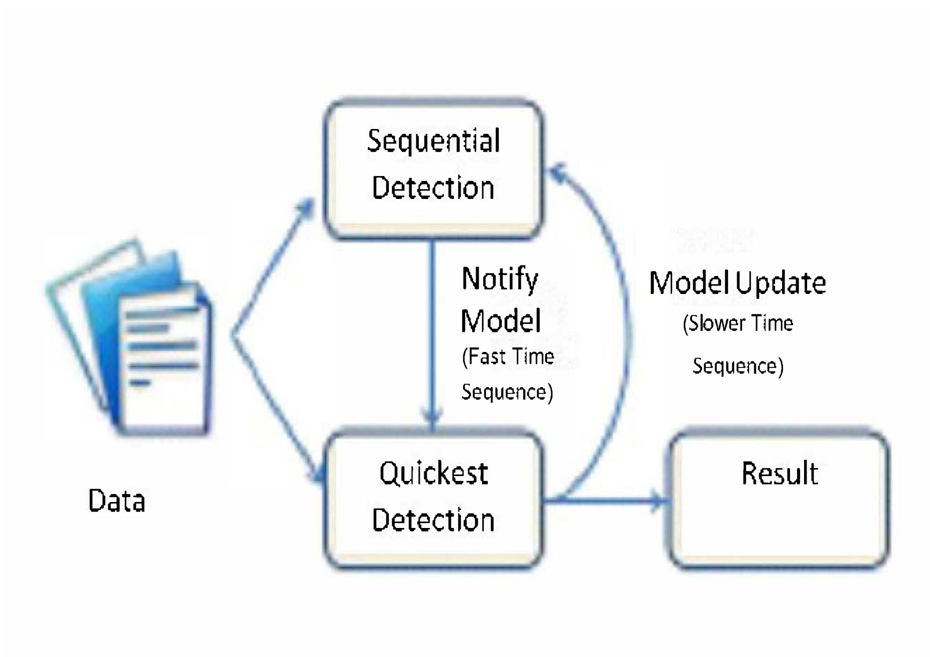

Figure 4.2: Composite Sequential Analysis and Quickest Detection scheme to determine sensor failure occurrence [Solaimani, 2014].

As Figure 4.2 suggests, a scheme that allows for the combination of various statistical testing methods, all working as a decision/detection process, would allow such a technique to acquire the data, analyze it accurately, and allow for verification purposes to occur. In this method it is envisioned that one analysis process would observe results from the other with the ability to stop the testing sequence on its own initiative.

Coupled with the aforementioned algorithmic enhancements, Trend Analysis and Optimal Stopping, such a scheme has the potential of informing on latent product failures and determining the operational state of the product at any given moment.

4.4 Conclusion

Through this thesis study, a functional comparison between a set of statistical analysis methods, Sequential Analysis and Quickest Detection, was conducted. These detection techniques are currently being implemented in sensor/product failure detection processes. Additionally, stand-alone simulation algorithms for each detection technique were developed and used to analyze a set of generated sensor models, a *strong sensor* and a *weak sensor* in the sense of their Signal-to-Noise-Ratio (SNR). These were used to compare similarities and dissimilarities between the two statistical methods, and develop an understanding of the functional operation of each as they could be deployed in failure recognition processes. Similarities between these two techniques include: their function of determining the system state being observed by minimizing delay and occurrence of false alarms, their utilization of *a posteriori* information to draw conclusions, and the ability for each technique to be adapted for various sensor model types. Dissimilarities between the aforementioned techniques include: the utilization techniques of the α-Knob component, the formulation each technique uses for interpreting the *a priori* information, and the variability that exists in the results from the in-depth analysis of each this thesis outlined.

This comparative study assists future researchers, who are unfamiliar with these techniques, towards developments of new and successful failure detection schemes. The detection techniques outlined in this thesis will aid in the field of failure recognition as it pertains to those processes that require the use of low-SNR sensing techniques because high-SNR sensors are either not available or too costly. These detection processes function accurately at the price of collecting a larger set of measurements, where the techniques discussed in this thesis characterize the delay-accuracy trade-off in such settings.

While investigating each method independently, developing some combination of the methods is suggested for future work. This combination technique could be further enhanced through the use of additional analysis techniques such as Trend Analysis methodologies and the theory of Optimal Stopping to decrease potential delay in the decision/detection processes and further enhance the accuracy of the detection procedure.

REFERENCES

[Alexander G. Tartakovsky and Sokolov, 2013] Alexander G. Tartakovsky, A. S. P. and Sokolov, G. (2013). Efficient computer network anomaly detection by changepoint detection methods. IEEE Journal of Selected Topics in Signal Processing, 7(1):4–11.

[Aminikhanghahi and Cook, 2017] Aminikhanghahi, S. and Cook, D. J. (2017). A survey of methods for time series change point detection. Knowl Inf Syst.

[Bai and Gupta, 2017] Bai, C.-Z. and Gupta, V. (2017). An on-line sensor selection algorithm for sprt with multiple sensors. IEEE.

[Basseville, 1988] Basseville, M. (1988). Detecting changes in signals and systems - a survey. International Federation of Automatic Control, 24(3):309–326.

[Braglia, 2000] Braglia, M. (2000). Mafma: multi-attribute failure mode analysis. International Journal of Quality and Reliability Management.

[Chopin, 2007] Chopin, N. (2007). Dynamic detection of change points in long time series. 59:349–366.

[Data-Science, 2020] Data-Science, P. (2020). Null hypothesis and alternative hypothesis explained in 2020. https://datasciencepr.com/null-hypothesis-and-alternative-hypothesis-explained-in-2020/. Data Science PR.

[Desmos, 2021] Desmos, C. (2021). Normal distribution cdf. (https://www.desmos.com/calculator/ssejz0yc7i).

[Dodge and Romig, 1929] Dodge, H. F. and Romig, H. G. (1929). A method of sampling inspection. The Bell System Technical Journal, 8(4):613–631.

[Gale, 2008] Gale, T. (2008). Sequential analysis. International Encyclopedia of the Social Sciences.

[Geng and Lai, 2013] Geng, J. and Lai, L. (2013). Bayesian quickest change point detection an localization in sensor networks. Global Conference on Signal and Information Processing, pages 871–874.

[Gladwin, 1997] Gladwin, L. A. (1997). Alan turing, enigma, and the breaking of german machine ciphers in world war ii.

[Glen, 2016a] Glen, S. (2016a). Mann kendall trend test: Definition, running the test. https://www.statisticshowto.com/mann-kendall-trend-test/.

[Glen, 2016b] Glen, S. (2016b). Seasonal kendall test. https://www.statisticshowto.com/seasonal-kendall-test/.

[Glen, 2016c] Glen, S. (2016c). Spearman rank correlation (spearman's rho): Definition and how to calculate it. https://www.statisticshowto.com/probability-and-statistics/correlation-coefficient-formula/spearman-rank-correlation-definition-calculate/.

[Heckert, 2015] Heckert, A. (2015). Cox stuart test. https://www.itl.nist.gov/div898/software/dataplot/refman1/auxillar/coxstuar.htm.

[Jeong, 2003] Jeong, A. C. (2003). The sequential analysis of group interaction and critical thinking in online threaded discussions. The Americal Journal of Distance Education, pages 25–43.

[Killick, 2014] Killick, R. (2014). Nonparametric methods for online changepoint detection.

[Krol, 2017] Krol, R. (2017). stoplight colormap. (https://github.com/lrkrol/stoplight-colormap). GitHub.

[Lai, 2001] Lai, T. L. (2001). Sequential analysis: Some classical problems and new challenges. pages 303–408.

[Leian Chen and Wang, 2018] Leian Chen, S. L. and Wang, X. (2018). Quickest fault detection in photovoltaic systems. IEEE Transactions on Smart Grid, 9(3):1835–1847.

[Leon, 2011] Leon, H. C. M. (2011). Lean product development reseach: Current state and future directions. 23(1):29–51.

[Lifeng Lai and Poor, 2008] Lifeng Lai, Y. F. and Poor, H. V. (2008). Quickest detection in cognitive radio: A sequential change detection framework. IEEE GLOBECOM 2008 - 2008 IEEE Global Telecommunications Conference, pages 1–5.

[Pan and Zheng, 2020] Pan, Y. and Zheng, Z. (2020). Bayesian online change point detection method for process monitoring. Chinese Control and Decision Conference, CCDC.

[Poor and Hadjiliadis, 2009] Poor, H. V. and Hadjiliadis, O. (2009). Quickest Detection. Cambridge University Press.

[Qiyue Zou and Sayed, 2010] Qiyue Zou, S. Z. and Sayed, A. (2010). Cooperative sensing via sequential detection. Transactions on Signal Processing, 58(12):6266–6283.

[Quality-One, 2020] Quality-One, I. (2020). Failure mode and effects analysis (fmea). https://quality-one.com/fmea/. Quality-One International.

[Richard J. Radke and Roysam, 2005] Richard J. Radke, Srinivas Andra, O. A.-K. and Roysam, B. (2005). Image change detection algorithms: A systematic survey. IEEE Transactions on Image Processing, 14(3).

[Robert B. Stone and Stock, 2005] Robert B. Stone, I. Y. T. and Stock, M. E. (2005). Linking product functionality to historic failures to improve failure analysis in design. Research in Engineering Design.

[Robert B. Stone and Wie, 2003] Robert B. Stone, I. Y. T. and Wie, M. V. (2003). The function-failure design method. Mechanical Design.

[Ronald E. Walpole and Ye, 2017] Ronald E. Walpole, Raymond H. Myers, S. L. M. and Ye, K. (2017). Probability & Statistics for Engineers & Scientists, volume Ninth Edition. Pearson.

[Rudolf B. Blazek and Tartakovsky, 2001] Rudolf B. Blazek, Hongjoong Kin, B. R. and Tartakovsky, A. (2001). A novel approach to detection of 'denial-of-service' attacks via adaptive sequential and batch-sequential change-point detection methods. IEEE.

[Sabato Santaniello and Sarma, 2011] Sabato Santaniello, Samuel P. Burns, A. J. G. J. M. S. W. S. A. and Sarma, S. V. (2011). Quickest detection of drug-resistant seizures: an optimal control approach. Epilepsy & Behavior, 22:S49–S60.

[Shaffer, 1996] Shaffer, J. (1996). Multiple hypothesis testing. Psychology, 46:561–84.

[Shewhart, 2017] Shewhart, W. A. (2017). Economic Control of Quality Of Manufactured Product. Princeton. Literature review revealed another researcher necessary for accurate historical representation leading up to development of Sequential Analysis.

[Shilpy Sharma and Obimbo, 2016] Shilpy Sharma, D. A. S. and Obimbo, C. (2016). Trend analysis and change point techniques: a survey. Joint Center on Global Change and Earth System Science of the University of Maryland and Beijing Normal University and Springer-Verlag.

[Siegmund, 2003] Siegmund, D. (2003). Herbert robbins and sequential analysis. The Annals of Statistics, 31(2).

[Sile Hu and Chen, 2018] Sile Hu, Qiaosheng Zhang, J. W. and Chen, Z. (2018). Real-time particle filtering and smoothing algorithms for detecting abrupt changes in neural ensemble spike activity. Innovative Methodology— Neural Circuits, 119:1394–1410.

[Sochman and Matas, 2005] Sochman, J. and Matas, J. (2005). Waldboost - learning for time constrained sequential detection. CVPR.

[Solaimani, 2014] Solaimani, M. (2014). Statistical technique for online anomaly detection using spark over heterogeneous data from multi-source vmware performance data. IEEE. Problem Graphic was adopted and adjusted for the specific needs of this research.

[SPSS, 2021] SPSS, T. (2021). Normal distribution - quick introduction. https://www.spss-tutorials.com/category/beginners/statistics-glossary/n/.

[Stephen R. Carpenter and Pace, 2014] Stephen R. Carpenter, William A. Brock, J. J. C. and Pace, M. L. (2014). A new approach for rapid detection of nearby thresholds in ecosystem time series. Oikos, 123:290–297.

[Taposh Banerjee and Veeravalli, 2014] Taposh Banerjee, Yu Christine Chen, A. D. D.-G. and Veeravalli, V. V. (2014). Power system line outage detection and identification - a quickest change detection approach. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pages 3450–3454.

[Taragay Oskiper, 2005] Taragay Oskiper, H. V. P. (2005). Quickest detection of a random signal in background noise using a sensor array. EURASIP Journal on Advances in Signal Processing, 2005(1):13–24.

[Wald, 1945] Wald, A. (1945). Sequential tests of statistical hypothesis. The Annals of Mathematical Statistics, 16(2):117–186.

[Wald, 2004] Wald, A. (2004). Sequential Analysis. Dover Publications, Inc.

[Williams, 2015] Williams, P. (2015). Product design and technology bangor dylunio cynnyrch a thechnoleg. WJEC Innovations Awards.

[Yi Huang and Han, 2016] Yi Huang, Jin tang, Y. c. H. L.-K. A. C. and Han, Z. (2016). Real-time detection of false data injection in smart grid networks: An adaptive cusum method and analysis. IEEE Systems, 10(2):532–543.

[Zhongxiang Dai and Jaillet, 2019] Zhongxiang Dai, Haibin Yu, B. K. H. L. and Jaillet, P. (2019). Bayesian optimization meets bayesian optimal stopping.

VITA

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